

# Der Blockplanungsansatz: Eine Fallstudienanwendung aus der Getränkeindustrie

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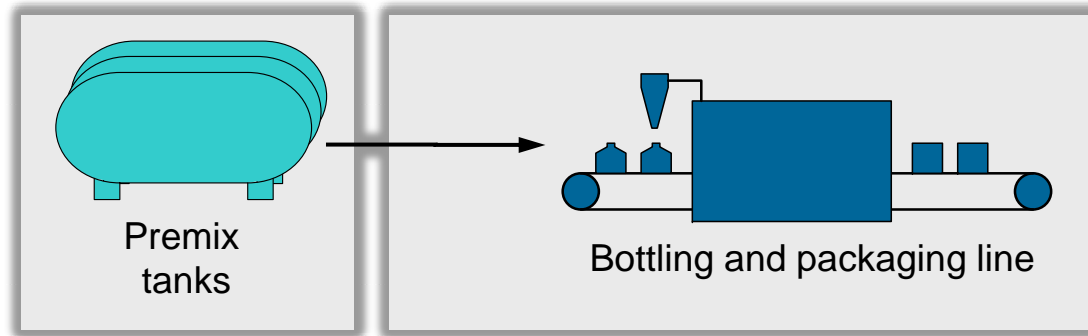
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## ➤ Outline

- Background: make-and-pack production
  - Block planning principle
  - Model formulation
  - Computational results
  - Conclusions
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# Make-and-pack production



- To be found in the **consumer goods industry**, e.g. in the production of food, beverages, detergents, cosmetics.
- There is often a **single bottleneck** stage, e.g. a combined final bottling and packaging stage.
- Often **highly capital-intensive production equipment** used, e.g. investment of € 20 Mio for a bottling and packaging line.

# Product-line assignment

## *Production of beverages*

*Production lines*

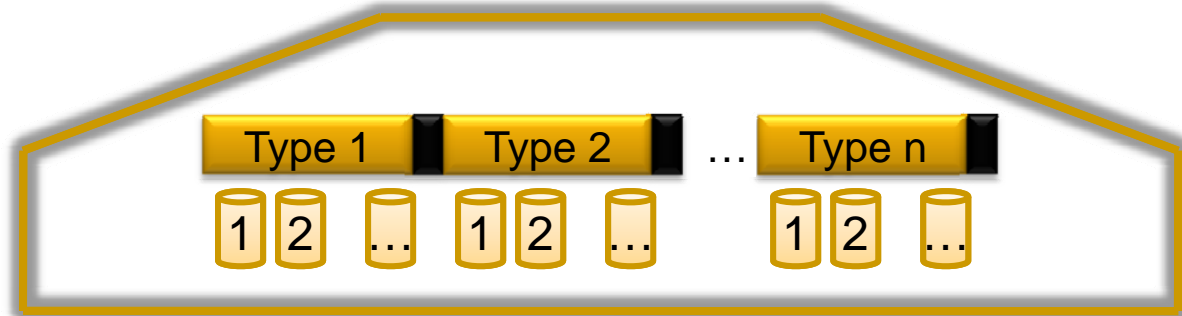
**Carton boxes**

**Plastic bottles**

**Glass bottles**

*Packaging form*

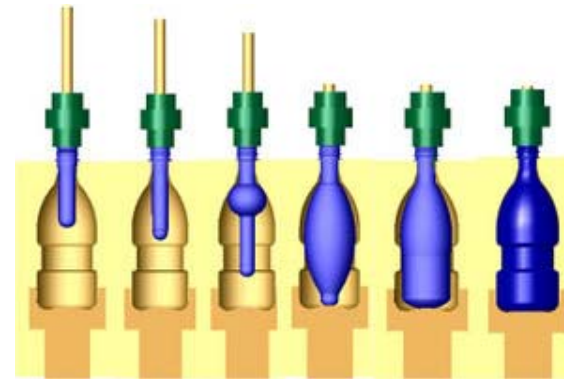
*Product type (recipe)*



# Setup drivers



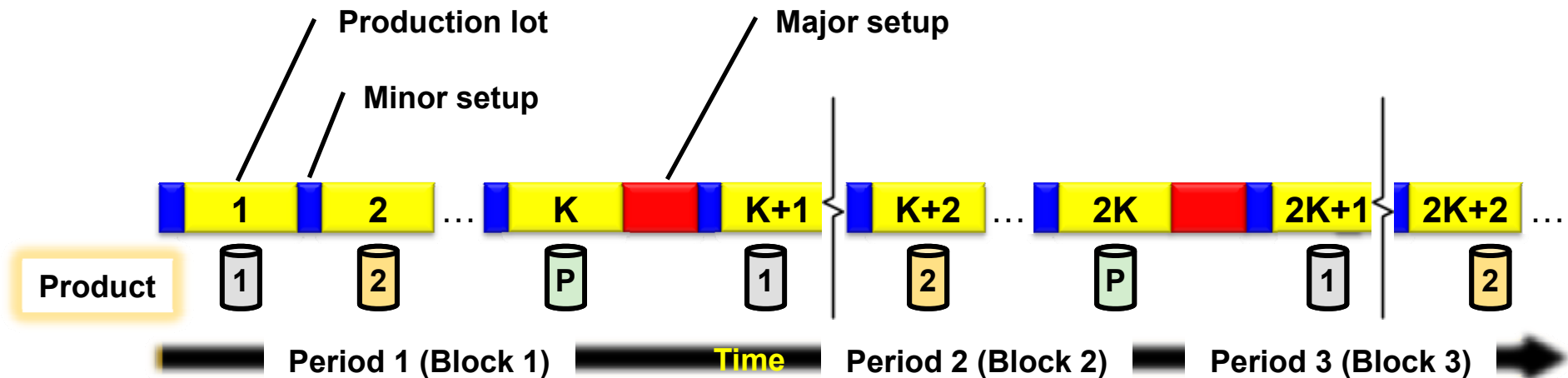
- **Major setup times** for changing the packaging format
  - e.g. 14 hours for setting up a stretch blow molding machine



- **Minor setup times** for changing the product type (recipe)
  - e.g. 90 minutes for cleaning pipelines and adjusting tanks

# Basic block planning principle

- Natural production sequence, e.g. from light to dark



- Establishment of **cyclical** production patterns
- One block per week with **flexible** start-off time

# Key issues of block planning

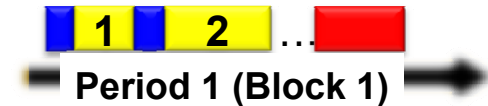


- Given **setup families** and fixed **setup sequences** within a family



- **Binary variables** for product setups and **variable lot sizes**

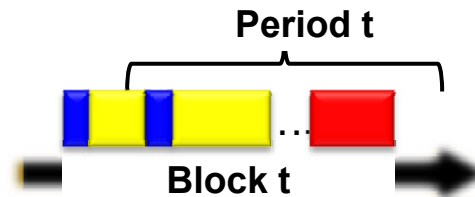
- Each **block** is assigned to a **macro-period**, e.g. a week.



- **Demand** is assigned to **micro-periods**, e.g. days.

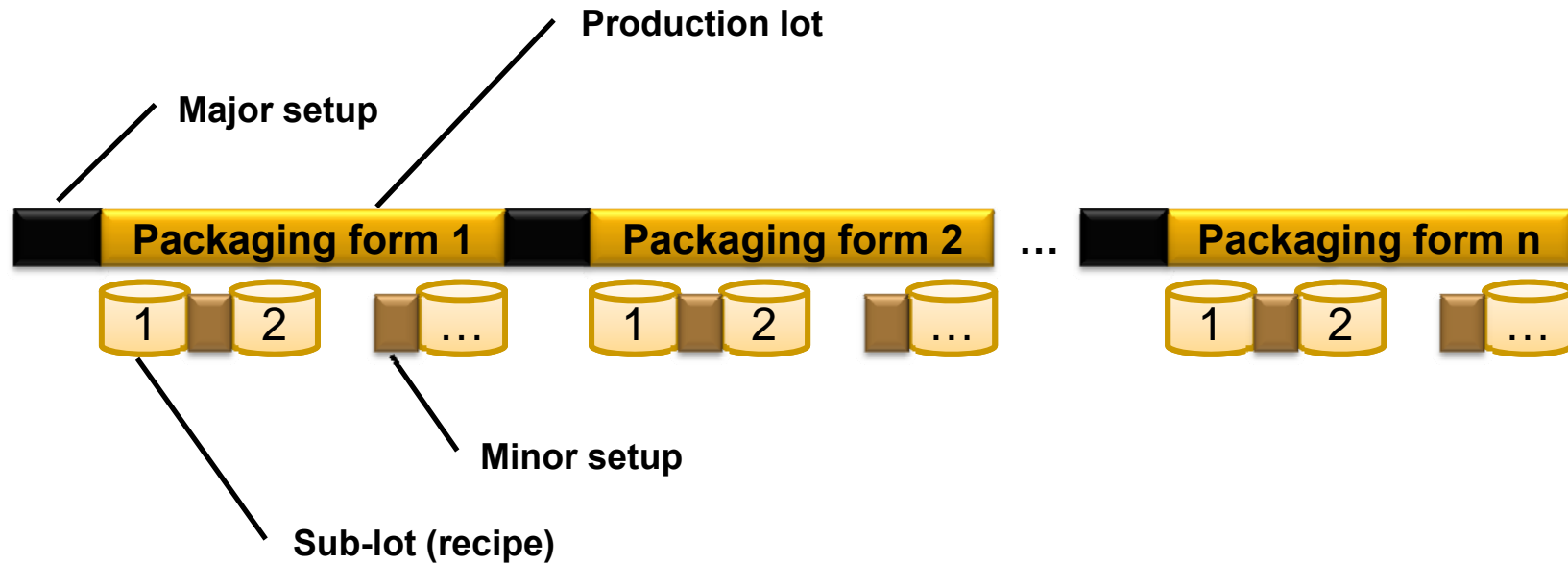


- The **start and ending times of the blocks are variable**, i.e. blocks may start in a previous period, but have to be completed before the end of the period they are assigned to.



- **Objective function**: Minimize total processing, setup and holding costs

# Block pattern for a bottling line



- Applications in consumer goods industry:
  - yoghurt production (cf. Lütke Entrup et al. 2005)
  - hair dye production (cf. Günther et al. 2006)



# MILP model formulation



$$\sum_{k \in K_{lt}} \rho_{kl} \leq \sigma_{lt} \cdot |K_{lt}|$$

Production lots only activated if the block is set up

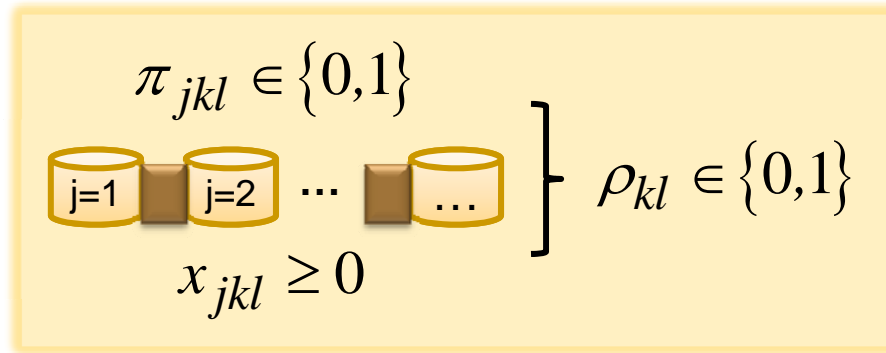
$$\forall l \in L, t \in T$$

Lot size enforced to zero if no setup takes place

$$X_{kl} \leq M_{kl} \cdot \rho_{kl}$$

$$\forall k \in K_l, l \in L$$

# MILP model formulation



$$\sum_{j \in J_{kl}} \pi_{jkl} \leq \rho_{kl} \cdot |J_{kl}|$$

Sub-lots only activated if the production lot is set up

$$\forall k \in K_l, l \in L$$

$$x_{jkl} \leq m_{jkl} \cdot \pi_{jkl}$$

Sub-lot size enforced to zero if no setup takes place

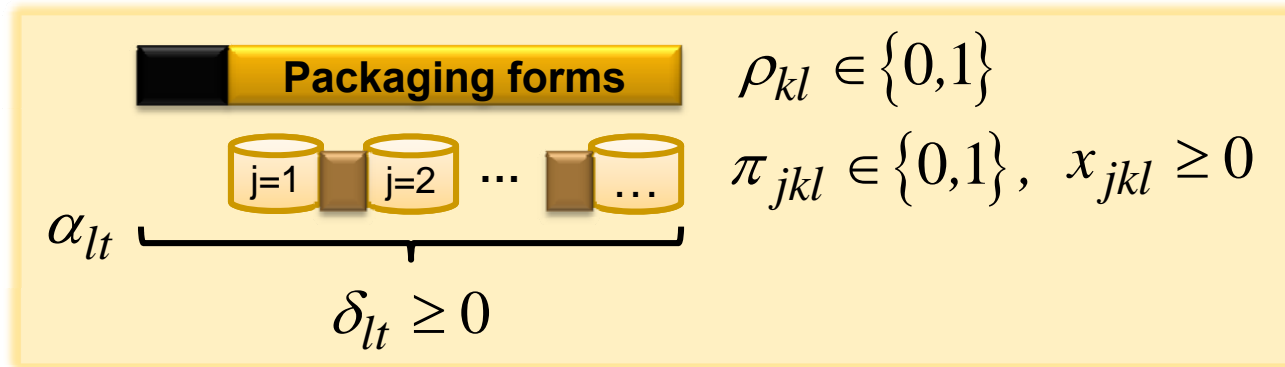
$$\forall j \in J_{kl}, k \in K_l, l \in L$$

$$\sum_{j \in J_{kl}} x_{jkl} = X_{kl}$$

Allocation of production lot size between sub-lots

$$\forall k \in K_l, l \in L$$

# MILP model formulation



$$\delta_{lt} = \sum_{k \in K_{lt}} \left( s_l \cdot \rho_{kl} + \sum_{j \in J_{kl}} (s_l \cdot \pi_{jkl} + a_{jkl} \cdot x_{jkl}) \right) \quad \text{Duration of a block}$$

$\forall l \in L, t \in T$

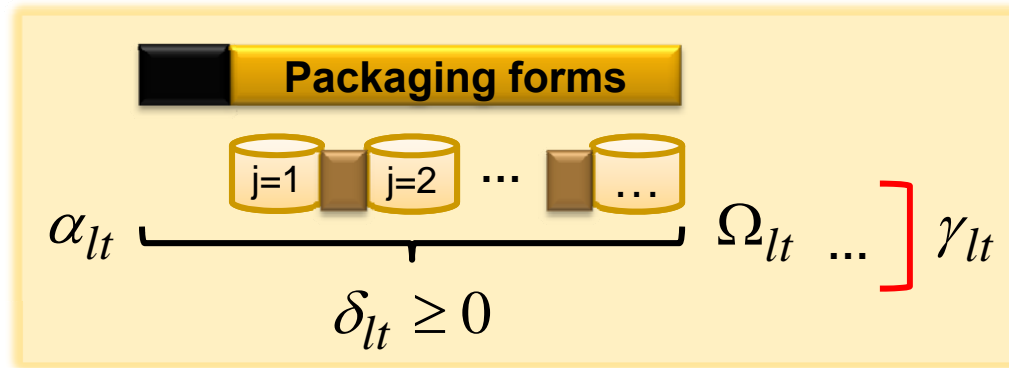
Start time of a block

$$\alpha_{lt} \geq \underline{\alpha}_{lt} \quad \forall l \in L, t \in T$$

Succession of blocks

$$\alpha_{lt} \geq \alpha_{l,t-1} + \delta_{l,t-1} \quad \forall l \in L, t = 2, \dots, |T| \text{ with } \alpha_{l1} = 0$$

# MILP model formulation



Blocks with production lots must be completed before the end of the week.

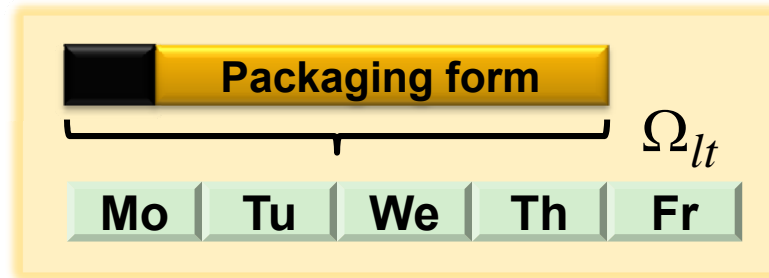
$$\alpha_{lt} + \delta_{lt} \leq \gamma_t \quad \forall l \in L, t \in T$$

End time of production lots relative to the start time of the block

$$\Omega_{kl} = \Omega_{k-1,l} + S_l \cdot \rho_{kl} + \sum_{j \in J_{kl}} (s_l \cdot \pi_{kl} + a_{jkl} \cdot x_{jkl})$$

$$\forall k \in K_{lt}, l \in L, t \in T \text{ with } \Omega_{0l} = \alpha_{lt}$$

# MILP model formulation

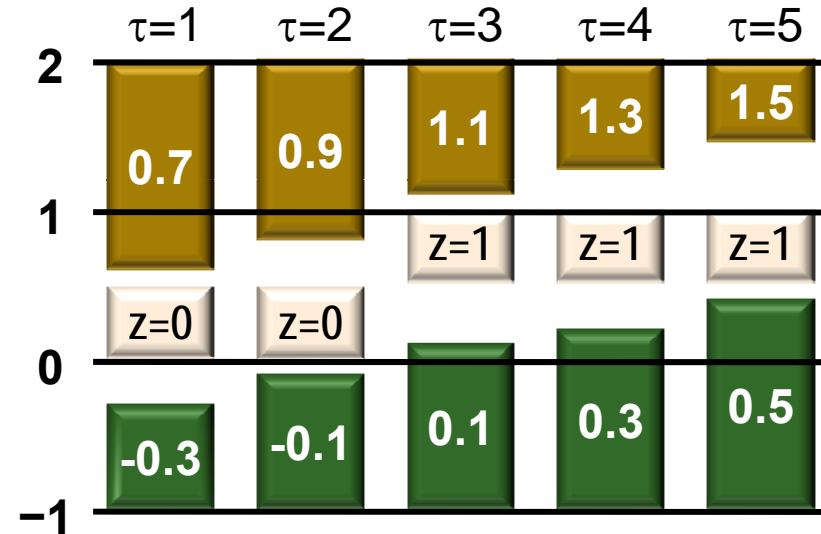


Heaviside function for tracing the daily completion of production lots

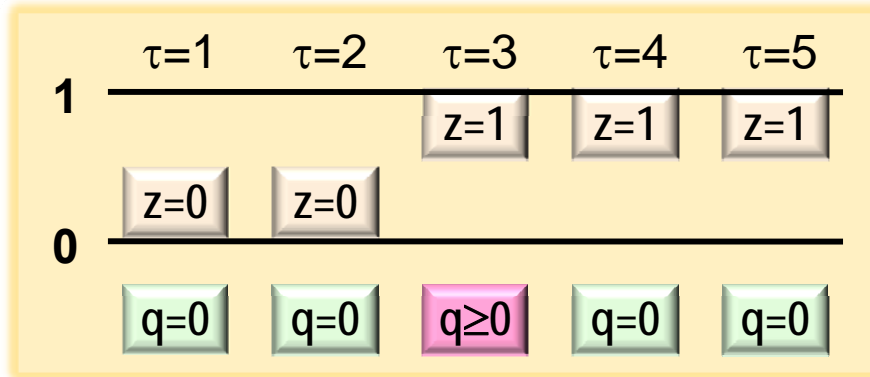
$$\frac{\tau - \Omega_{kl}}{d_{kl}} \leq z_{kl\tau} \leq 1 + \frac{\tau - \Omega_{kl}}{d_{kl}}$$

$$\forall k \in K_l, l \in L, \tau \in D_{kl}$$

➤ Example:  $\Omega = 2.5$ ;  $\tau = 1, \dots, 5$



# MILP model formulation



Production output achieved when the heaviside variable switches from 0 to 1

$$\sum_{j \in J_{kl}} q_{jl\tau} \leq M_{kl} \cdot (z_{kl\tau} - z_{kl,\tau-1}) \quad \forall k \in K_l, l \in L, \tau \in D_{kl} \text{ with } z_{kl\tau} = 0$$

$$\sum_{\tau \in D_{kl}} q_{jl\tau} = x_{jkl} \quad \forall j \in J_{kl}, k \in K_l, l \in L$$

Inventory balances for each product and day

$$F_{p\tau} = F_{p,\tau-1} + \sum_{l \in L_p} \sum_{j \in J_{pl}} q_{jl\tau} - E_{p\tau} \quad \forall p \in P, \tau \in D \text{ with } F_{p0} = \text{given}$$

# MILP model formulation



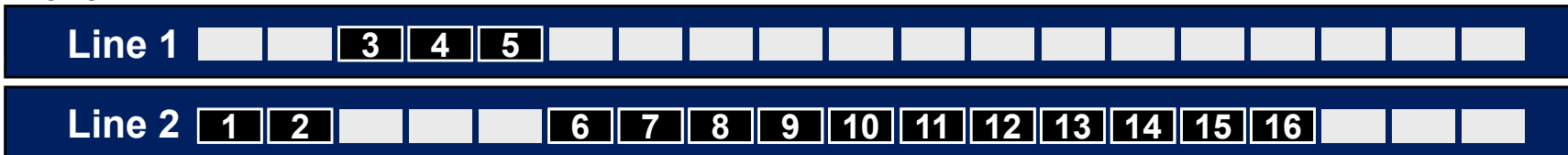
Objective function: Minimize total costs for production, clean-outs, major and minor setups, and inventory holding

$$\min \left[ \sum_{l \in L} \sum_{t \in T} c_l^{Prod} \cdot \delta_{lt} + \sum_{l \in L} \sum_{t \in T} c_l^{Clean} \cdot \sigma_{lt} + \sum_{l \in L} \sum_{k \in K_l} c_l^{Maj} \cdot \rho_{kl} + \right. \\ \left. \sum_{l \in L} \sum_{k \in K_l} \sum_{j \in J_{kl}} c_l^{Min} \cdot \pi_{jkl} + \sum_{p \in P} \sum_{\tau \in D} c_p^{Inv} \cdot F_{p\tau} \right]$$

# Computational Experiments

- ## ➤ Product-line assignments in the FRUTADO case study

## Plant 1



## Plant 2



### Plant 3



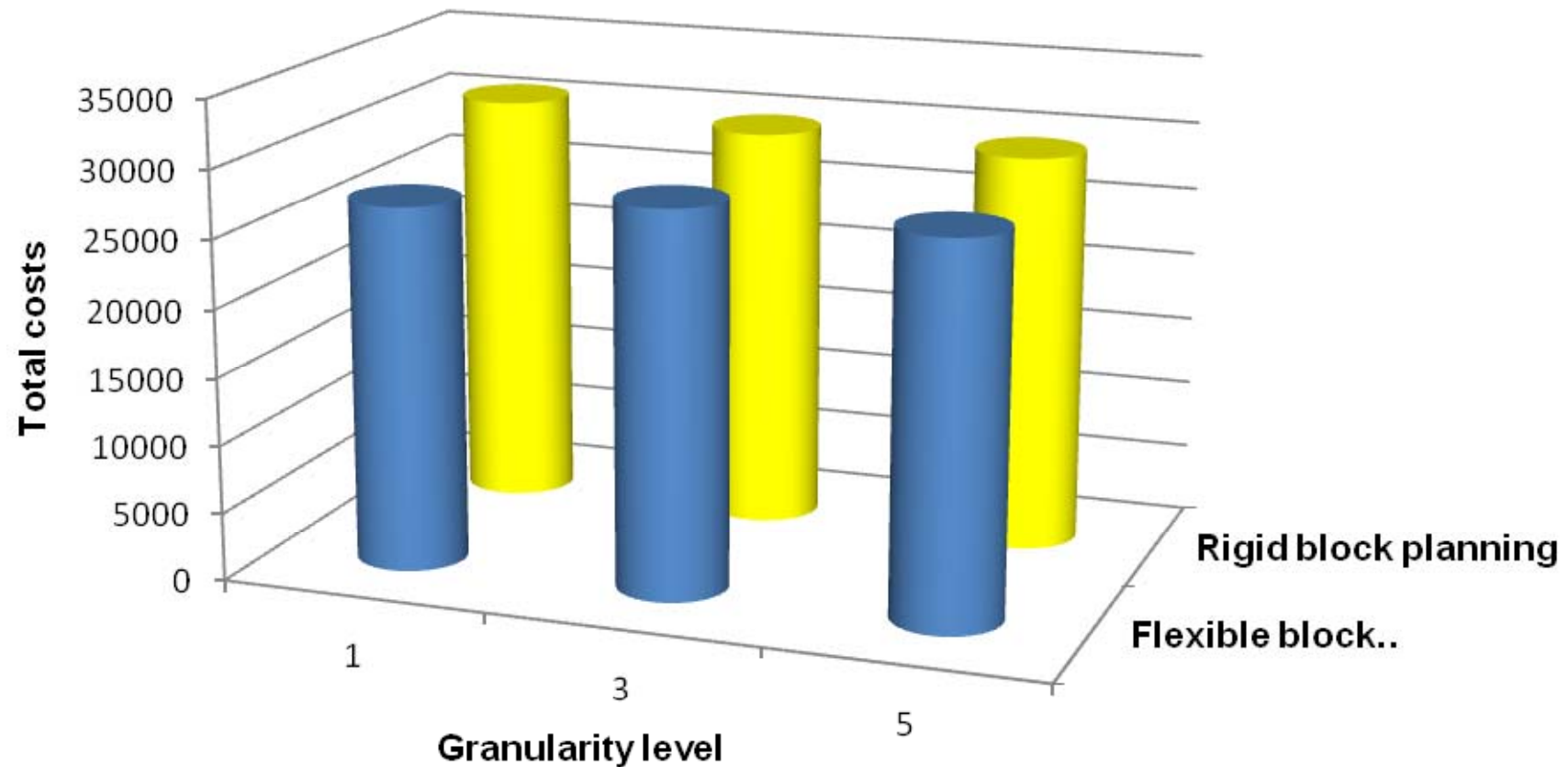


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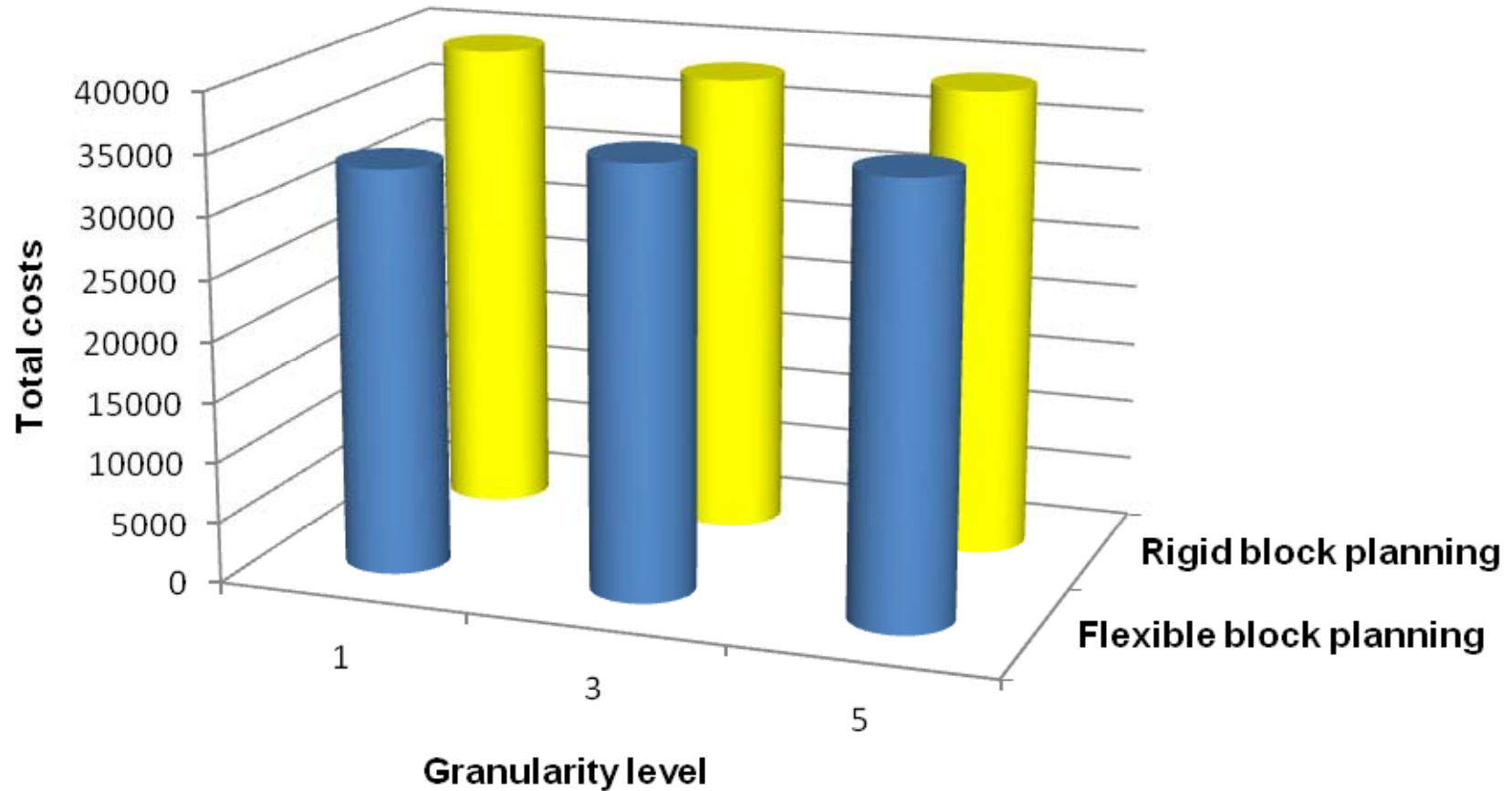
# Experimental setting

- Implementation using ILOG OPL Studio 6.1.1 with CPLEX 11.2
  - PC: Dual Xeon Quad Core 2.5 GHz processor and 4 GB RAM
  - Two **capacity load scenarios**: 75% and 90%
  - Three **demand granularities**: 1 (low), 3 (medium) ,5 (high)
  - **Daily demand** randomly assigned to DCs
  - All other data taken from the **FRUTADO case**
- Comparison of rigid vs. flexible block planning, i.e. one-week vs. two-week time window per block
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## Numerical results: 75% capacity load scenario



## Numerical results: 90% capacity load scenario



# CPU times for an extended model formulation

## ➤ 75% capacity load

Granularity level	CPU time (flexible / rigid block planning) in sec	
	min	max
1	4 / 5	7 / 358
3	10 / 9	102 / 88
5	10 / 16	69 / 109

## ➤ 90% capacity load

Granularity level	CPU time (flexible / rigid block planning) in sec	
	min	max
1	3 / 8	6 / 1991
3	16 / 38	56 / 1080
5	70 / 94	983 / 560

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# Conclusion

- To support block planning in a make-and-pack environment a novel MILP model formulation based on a continuous representation of time has been developed.
  - The model formulation exploits human expertise on the definition of setup families and the sequence of production runs within a block.
  - The model considers the daily assignment of demand elements.
  - Flexible block planning appears to be superior compared to rigid block planning.
  - MILP modelling provides a flexible framework to integrate many application-specific features.
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