

83th Meeting of the GOR Working Group

## Practice of the Mathematical Optimization

### **Mathematical Optimization in People Transportation – Column Generation in Airlines, Rail & Bus Optimization Systems –**

19–20 November, 2009

Physikzentrum, Bad Honnef, Germany  
([www.pbh.de](http://www.pbh.de))

### Organisation

Josef Kallrath & Alexander Lavrov  
GOR AG „Praxis der mathematischen Optimierung“

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# Mathematical Optimization in People Transportation

## – Column Generation in Airlines, Rail & Bus Optimization Systems –

The Transportation industry sector comprises a wide range of planning and control services, covering different modes of transport, such as airline, public transport, and railway. The market is dynamically changing. Globalization and technical progress created double digit revenue growth in the industry in the early part of the 21st century, however the economic downturn will result in a dramatic drop in the sector's growth in 2009.

Solving real-world scheduling problems is a complicated task with increasing demands on performance and flexibility. A few commercial software packages for addressing scheduling problems in the transportation industry exist. Advanced systems are based on tailored column generation algorithms for solving large scale nonlinear network optimization problems.

This workshop will give an overview about the state-of-the art technologies illustrated by real-world applications. Particular attention will be paid to contributions from industry and industry research solving the problems in their field using column generation techniques and related methods, such as branch-and-price and constrained shortest path methods.

In 15 talks, each approx. 30-45 minutes, experts from practice, research institutions or software companies, will present selected problems and the corresponding solutions. Confirmations for their talks have been obtained from the following speakers:

B. Amberg and Michael Beck (University Paderborn and initplan GmbH, Germany)  
*Integrated Vehicle and Crew Scheduling with Time Windows for Scheduled Trips in Public Transport*

Dr. Oliver Bastert (FICO, Munich, Germany)  
*Solution Techniques for Airline Optimization Problems*

Dr. Ralf Borndörfer (Löbel, Borndörfer & Weider GbR and Konrad Zuse Institute, Berlin, Germany)  
*Multicriteria Optimization in Public Transportation*

Dr. Michael Bussieck (GAMS Software GmbH, Köln, Germany)  
*Column Generation in GAMS*

Andre De Stefano (Zentrum für Integrierte Verkehrssysteme, Darmstadt, Germany)  
*Scheduling of School Buses*

Sven de Vries (Universität Trier, Trier)  
*Branch-and-Price for Combinatorial Slot Auctions*

Guido Diepen (AIMMS, Paragon Decision Technology, The Netherlands)  
*Planning of buses and gates at Amsterdam Airport Schiphol*

Ivan Dovica (Konrad Zuse Institute, Berlin, Germany)  
*An Integer Programming Approach to Robust Tail Assignment*

Armin Fügenschuh (Zuse Institut Berlin, Berlin, Germany)  
*Scheduling of School Buses*

Thomas Hanne, (University of Applied Sciences Northwestern Switzerland, Olten, CH)  
*Optimization Approaches in Airline and Railway Planning - A Comparative Survey*

Henning Homfeld (TU Darmstadt, Darmstadt, Germany)  
*Routing Cars in Rail Freight Service*

Prof. Dr. Knut Haase (TU Dresden, Institut für Wirtschaft und Verkehr, Dresden, Germany)  
*Annual Holiday Planning for the Crew of a Public Transport Company*

Dr. Marco Lübbecke (TU Berlin, Berlin, Germany)  
*Branch-and-Price for a Split Pickup and Split Delivery Problem*

Prof. Dr. Karl Nachtigall (TU Dresden, Faculty of Transport and Traffic Sciences, Dresden, Germany)  
*A Pre-tactical Generalised Air Traffic Flow Management Problem*

Dr. habil. Ivo Nowak (Lufthansa Systems GmbH, Berlin, Germany)  
*Column Generation in Airline Planning & Control*

Daniel Potthoff (Erasmus School of Economics, Erasmus University Rotterdam, The Netherlands)  
*Railway Crew Rescheduling*

Steffen Rebennack (University of Florida, Industrial & Systems Engineering, FL)  
*Branch and Price Algorithm for Multimodal Evacuation Problems*

Thomas Schlechte (Konrad Zuse Institute, Berlin, Germany)  
*Branch and Price for the Train Tabling Problem*

Thomas Winter (Lufthansa Systems Berlin GmbH, Berlin)  
*Solution Techniques for Revenue Optimization*

In particular, the following thematic fields will be addressed:

- Column generation methods
- Robust and integrated optimization
- Applications in airline, public transport, and railway

We expect an interesting overview of „Mathematical Optimization in People Transportation“, reflecting modern requirements, possibilities, and limitations. Part of the official program is a visit and a guided tour through the romanic double church St. Maria und St. Clemens in Schwarzrheindorf (13:15-14:30) and Cologne Airport on Thursday, Nov 19, 2009 from 15:15-16:45.

The official opening is at 09:10 on 19.11.2009. The conference dinner takes place at 20:00 on the same day. The closure of the event is scheduled for 20.11.2009, at about 18:00.

Further contributions are welcome if they describe techniques or problems relevant for the transportation industry. In particular, we look forward for practice-oriented application examples and case studies where optimization problems have been solved rigidly using column generation.

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83. Sitzung der GOR Arbeitsgruppen  
Praxis der Mathematischen Optimierung & Logistik und Verkehr

**Mathematical Optimization in People Transportation  
– Column Generation in Airlines, Rail & Bus Optimization Systems –**

Physikzentrum, Bad Honnef, November 19 & 20, 2009

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Thursday, Nov. 19 - 2009 : 09:10 – 23:00

09:10-09:20 **Opening and Welcome Session** (J. Kallrath, I. Nowak & K. Haase)

09:20-09:35 **Victor Gomer** Physikzentrum, Bad Honnef, Germany  
*Information on the Conference Center*

09:40-10:30 **Ivo Nowak** Lufthansa Systems GmbH, Berlin, Germany  
*Column Generation in Airline Planning & Control*

10:35-11:25 **Guido Diepen** AIMMS, Paragon Decision Technology  
*Planning of Buses and Gates at Amsterdam Airport Schiphol*

11:30-12:40 ----- Lunch Break -----

12:45-17:30 -----Visit & Guided Tour: Double-Church Schwarzerheindorf -----  
Cologne Airport

17:30-18:05 **Knut Haase** TU Dresden, Dresden, Germany  
*Annual Holiday Planning for the Crew of a Public Transport Company*

18:05-18:40 **Michael Bussieck** GAMS Software GmbH, Cologne, Germany  
*Column Generation in GAMS*

18:40-19:20 **Oliver Bastert** FICO, Munich, Germany  
*Solution Techniques for Airline Optimization Problems*

19:20-19:50 **Thomas Winter**, (Lufthansa Systems Berlin GmbH, Berlin)  
*Solution Techniques for Revenue Optimization*

19:50-20:00 **Internal Meeting** of the Working Group “Logistik und Verkehr”

20:00 - **Conference Dinner** – Buffet; get-together in the wine-cellar  
*Celebrating the 83<sup>th</sup> Meeting of our GOR Working Group*

Friday, Nov. 20 - 2009 : 08:50 – 17:30

- 08:50-09:30 **Dr. Marco Lübbecke** (TU Berlin, Berlin, Germany)  
*An overview of the Use of Branch-and-Price in Public Transportation Problems*
- 09:35-10:05 **Henning Homfeld** (TU Darmstadt, Darmstadt, Germany)  
*Routing Cars in Rail Freight Service*
- 10:10-10:40 **Daniel Potthoff** (School of Economics, Erasmus University Rotterdam)  
*Railway Crew Rescheduling*
- 10:40-11:00 ----- Coffee Break -----
- 11:00-11:30 **Ivan Dovica** (Konrad Zuse Institute, Berlin, Germany)  
*An Integer Programming Approach to Robust Tail Assignment*
- 11:35-12:05 **Thomas Schlechte** (Konrad Zuse Institute, Berlin, Germany)  
*Branch and Price for the Train Timetabling Problem*
- 12:10-12:40 **Karl Nachtigall** TU Dresden, Dresden, Germany  
*A Pre-tactical Generalised Air Traffic Flow Management Problem*
- 12:45-13:45 ----- Lunch Break -----
- 13:45-14:20 **Thomas Hanne**, (Uni Applied Sci. Northwestern Switzerland, Olten, CH)  
*Optimization Approaches in Airline and Railway Planning - A Comparative Survey*
- 14:20-14:55 **Steffen Rebennack** (Univ. of Florida, Gainesville, FL)  
*Branch and Price Algorithm for Multimodal Evacuation Problems*
- 14:55-15:30 **B. Amberg** (University Paderborn, Germany)  
**Michael Beck** (initplan GmbH, Karlsruhe, Germany)  
*Integrated Vehicle and Crew Scheduling with Time Windows for Scheduled Trips in Public Transport*
- 15:30-15:45 ----- Coffee Break -----
- 15:45-16:25 **Ralf Borndörfer** (Löbel, Borndörfer & Weider GbR / Zuse Institute, Berlin)  
*Multicriteria Optimization in Public Transportation*
- 16:30-17:15 **Andre De Stefano** (Zentrum für Integrierte Verkehrssysteme, Darmstadt)  
**Armin Fügenschuh** (Konrad Zuse Institut Berlin, Berlin, Germany)  
*Scheduling of School Buses*
- 17:15-17:50 **Sven de Vries**, (Universität Trier, Trier)  
*Branch-and-Price for Combinatorial Slot Auctions*
- 17:50-18:00 **Final Discussion – End of the Workshop**

## The Speakers

**Boris Amberg** is a PhD candidate in the faculty of business administration and economics at the University of Paderborn, Germany. He is a graduate student at the International Graduate School of Dynamic Intelligent Systems and is supervised by Jun.-Prof. Dr. Natalia Kliewer. Boris Amberg got his diploma degree in 2007 in business computing from the University of Paderborn.

**Oliver Bastert** is a Lead Engineer with FICO. He has 15 years experience in developing optimization algorithms and modeling optimization tasks from a number of different industries. He holds a doctorate degree in mathematics from the Munich Technical University, Germany.

**Michael Beck** is Director Development at initplan GmbH a subsidiary of Init AG, Germany since 2008. From 1988 to 2008 he was employed by PTV AG, Germany, where he was responsible for the development of planning and optimization software for public transport companies.

**Ralf Borndörfer** is co-head of the Zuse Institute Berlin's optimization department. He has participated in the development of optimizers for airline crew scheduling, bus and train driver scheduling, vehicle scheduling, line planning, and railway track allocation, several of which are now used within commercial scheduling systems. His research interests lie in integer programming and combinatorial optimization with applications to traffic and transport. Ralf received his Ph.D. from Technical University Berlin, Germany.

**Michael R. Bussieck** is a Senior Research Analyst at GAMS Software GmbH. From 1999 to 2004 he worked at the GAMS Development headquarters in Washington DC, USA. He received his Ph.D. from Technical University Braunschweig, Germany.

**André De Stefano** studied civil engineering at the University of Applied Sciences in Cologne (Diploma in 2006) and in Bratislava (Master of Engineering in 2007). Since 2008 he is head of the traffic planning division at the Zentrum für Integrierte Verkehrssysteme (Center for Integrated Traffic Systems) ZIV in Darmstadt. He is mainly interested in planning public transport systems and the application of OR based optimization methods.

**Sven de Vries** is professor of Operations Research (math. dept.) at Universität Trier. He studied mathematics at the Universität Hamburg and Ohio State University and completed his PhD and habilitation at TU München. After holding an acting professor's appointment at TU München and being visiting professor at Yale (econ. dept.) he held for two years the OR chair at Groningen University. His interests are Combinatorial Optimization and Auctions.

**Guido Diepen** works at Paragon Decision Technology where he is an AIMMS specialist, helping customers getting the most out of the AIMMS mathematical modeling system. Guido received his PhD in Computer Science from Utrecht University (NL) for work on column generation algorithms for machine scheduling and integrated airport planning.

**Ivan Dovica** is a PhD candidate in Discrete Optimization at Zuse Institute Berlin supervised by Prof. Dr. Dr. h.c. mult. Martin Grötschel and Dr. Ralf Borndörfer. He received his diploma in Computer Science in 2006 from Charles University in Prague, Czech Republic. Currently, is working on project Robust Tail Assignment. His research interests are in large-scale optimization, airline optimization and column generation.

**Armin Fügenschuh** received his diploma degree in mathematics and computer sciences from

Oldenburg University in 2000. He was PhD candidate at Technische Universität Darmstadt, where he received his PhD in 2005. After that he became member of a collaborative research center (SFB) as a postdoc researcher. Since April 2009 he is a postdoc at the Zuse Institute in Berlin (ZIB) in the Department for Optimization. His research interests include mixed-integer linear and nonlinear programming and its applications to real-world problems in logistic, transportation, and energy distribution.

**Knut Haase** is owner of the chair of business administration, transport and logistics at the institute for transport and economics at the technical university of Dresden. His main research activities are focused on concepts and quantitative methods for managing transport enterprises, i.e. analysis and optimization of economical structures and processes in transportation.

**Thomas Hanne** received a masters degree in Economics from Ruhr-University Bochum, a masters degree in Computer Science from University Dortmund, and a PhD in Economics from FernUniversity Hagen. From 1999 to 2007 he worked at the Fraunhofer Institute for Industrial Mathematics (ITWM) in the Department of Optimization as senior scientist. Since then he is Professor for Information Systems at the University of Applied Sciences Northwestern Switzerland. Thomas Hanne is author of about 40 journal and conference articles. His current research interests include multicriteria decision analysis, evolutionary algorithms, metaheuristics, scheduling, discrete-event simulation, logistics, and supply chain management.

**Henning Homfeld** is a PhD candidate at Technische Universität Darmstadt at the Chair of Discrete Optimization. His advisors are Prof. Dr. Alexander Martin and Dr. Armin Fügenschuh (Zuse Institute Berlin). Henning Homfeld got his diploma degree in 2007. He is interested in mixed-integer linear programming and its applications to real-world problems, mainly in the area of strategic planning in public transportation.

**Rainer Kaufhold** worked from 1992 until 1997 as scientific assistant at the Department for Flight Mechanics and Control at the University of Technology, Darmstadt Germany. In 1998 he received his Ph.D. in engineering by a thesis on *Design of ergonomic perspective terrain representations for cockpit*. In 1998 Rainer Kaufhold joined the research and development department of DFS Deutsche Flugsicherung. Since then he has been focusing his work on improving co-operative planning processes in ATM. He currently is the project manager of the nationally funded project K-ATM (Cooperative Air Traffic Management).

**Marco Lübbecke** is an assistant professor at TU Berlin's Institute of Mathematics. He received his Ph.D. in mathematics from TU Braunschweig in 2001, and finished his Habilitation in 2007 at TU Berlin. Marco's research interests are in integer programming and combinatorial optimization. It is not the exception but the rule that his results find their way into industrial practice as well.

**Karl Nachtigall** is chairholder of the Chair Traffic Flow Science in the Department of Logistics and Aviation at the Technical University of Dresden. He studied mathematics at the University of Hanover and received his PhD in Operations Research from the University of Hannover in 1989. Subsequently he conducted research for the DLR (Deutsches Zentrum für Luft- und Raumfahrt e. V.) in Brunswick as scientific staff in several scientific projects and received his postdoctoral lecture qualification from the University of Hildesheim in 1998.

**Ivo Nowak** is a Product Manager for operations research solutions at Lufthansa Systems Berlin GmbH (LSB) since 2004. He received his Ph.D. in 1994 from the Technical University Berlin, and his Habilitation in 2004 from the Humboldt University Berlin, where he worked



as an assistant professor. He is author of a book and a software package for mixed integer nonlinear programming. Currently he is involved in the development of an optimization suite for airline planning and control at LSB.

**Daniel Potthoff** is a PhD student at the Erasmus School of Economics. His research project on robust planning and re-scheduling in public transport is carried out in cooperation with the Department of Logistics of Netherlands Railways.

**Steffen Rebennack** is a PhD candidate in Industrial & Systems Engineering department at the University of Florida, USA. Prof. Dr. Pardalos is his thesis PhD adviser. Steffen Rebennack got his diploma degree in 2006 in mathematics from the University of Heidelberg, Germany. His research interests are in global optimization, modeling, decomposition methods, power systems modeling, power systems optimization and combinatorial optimization.

**Thomas Schlechte** is a PhD candidate in Optimization department at the Zuse Institute Berlin, Germany. Prof. Dr. Dr. h.c. mult. M. Grötschel and Dr. R. Borndörfer are his thesis PhD advisers and project heads in a research project *Slot Allocation for Railways*, supported by the Federal Ministry of Economics and Technology (BMW), grant 19M4031A. Thomas Schlechte got his diploma degree in 2004 in Applied Mathematics from Berlin university of technology (TU Berlin), Germany. His research interests are in linear and integer optimization, sophisticated solution approaches to large-scale problems, in particular for traffic applications.

**Thomas Winter** is a Senior Scientific Analyst at Lufthansa Systems Berlin GmbH (LSB) since 2008. Before he joined the revenue management team at LSB, from 2000 until 2007 he worked for Siemens Mobile Networks and Nokia Siemens Networks as Systems Engineer for Mobile Network Optimization and as Project Manager. He received his Ph.D. from Technical University Braunschweig, Germany.

# Integrated Vehicle and Crew Scheduling with Time Windows for Scheduled Trips in Public Transport

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In public bus transportation the two planning tasks vehicle scheduling and crew scheduling are traditionally solved sequentially with the implicit understanding that the scheduled time for timetabled trips remains fix. In order to improve cost efficiency two concepts have been developed over the last years: To get more flexibility while scheduling crews, vehicle and crew scheduling problems are tackled simultaneously; To extend flexibility while scheduling vehicles, variable trip departure and arrival times are considered. Obviously the combination of both concepts promises the largest savings, but likely leads to bursting computational times due to growing problem complexity.

In this work we combine both concepts by extending the integrated vehicle and crew scheduling problem with the possibility to shift scheduled trips within defined time windows.

We present and compare different variants of time window consideration in the resource planning process. The underlying network model of our approach bases on a time-space network structure. The integrated problem is solved with column generation in combination with Lagrangian relaxation. For the column generation pricing problem we use a special task-based model with levels, which enables the use of fast heuristics. Further, to reduce computational times we determine in a heuristic way a subset of trips critical for costs of schedule operations - shifting these trips is likely to lead to savings in the number of vehicles and crew duties and/or operational costs.

We examine the tradeoffs between solution quality and computational time of different variants of our solution approaches.

# Solution techniques for airline optimization problems

Oliver Bastert

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Airlines have to deal with a variety of large scale optimization problems, such as crew scheduling (pairing and rostering), aircraft scheduling, yield management and flight scheduling.

We will have a closer look at some of these problems, examine why they are difficult and discuss what can be done in order to solve them efficiently.

The FICO Xpress team (Dash Optimization before January 2008) has gained thorough experience in the field of airline optimization over the last decade and incorporated their knowledge into the FICO Xpress linear and mixed integer programming solver. We give an overview of the techniques that are particularly useful in this context.

FICO Xpress is widely used in the aviation industry. Jeppesen included Xpress as the underlying optimization engine within their applications and American Airlines uses Xpress as their optimizer and modeling environment.

# Multicriteria Optimization in Public Transportation

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Costs, operational stability, and employee satisfaction are typical objectives in optimization problems in public transportation. These criteria are traditionally simply merged into a single objective. In order to study the tradeoffs between competing goals, however, one needs to compute the entire Pareto curve. The talk discusses extensions of Lagrangean relaxation and column generation approaches to compute such Pareto curves for vehicle and crew scheduling problems in public transit.

# Column Generation in GAMS

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In transportation problems *path* oriented formulation are often preferred over *edge* based models. One of the reasons is that complex constraints can be embedded in the path variables. Column generation (CG) and Branch-and-Price (B&P) methods are **the** algorithmic choice for such problems. While several (mostly academic) software packages like MINTO, ABACUS, and BCP exist, there are only a few successful applications of path oriented model and CG/B&P in place. One reason for this is the level of expertise in various fields required: application knowledge, algorithmic/mathematical knowledge, and IT/CS knowledge. Such an investment requires either a large user base (e.g. vehicle routing) or an important and specialized application (e.g. aircraft and crew scheduling). In spite of its potential, the CG/B&P paradigm has not significantly penetrated the general mathematical programming (MP) modeling world.

The General Algebraic Model System (GAMS) has pioneered the dissemination of new MP technology over the last 20 years: equilibrium models, complementarity problems, stochastic optimization, and global optimization to name a few. Together with its rapid prototyping capability, GAMS might prove a good vehicle to advance the dissemination of path based models and CG/B&P. We present the general concept of how to implement path based models and some extensions of the widely used GAMS/BCH Facility that provide the different *hooks* required for a CG/B&P algorithm. The user *only* needs to fill in the problem specific part in the name space of the application.

# Scheduling of School Buses

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In many rural areas, the public bus service is demand-oriented: By far the biggest group of customers are pupils who are transported to their schools within certain strict time limits. Usually, all schools start around the same time, which causes a morning peak in the number of deployed buses. However, schools are allowed to change their starting times within some interval. The question is, how to simultaneously rectify the starting times for all schools and bus trips in a certain county so that the number of scheduled buses is minimal.

From a mathematical point-of-view, this problem is hard since it is a vehicle scheduling problem with coupled time windows, where additional coupling constraints on the time windows are introduced. To obtain feasible solutions we use a heuristic method, that can be embedded in a branch-and-cut framework as well as a column generation approach.

From the application's point-of-view, this problem is hard since it requires to gather reliable and error-free data in an environment where people usually work with pen-and-paper. Further it means to negotiate a lot with stakeholders from various parties having a large variety of interests, such as bus company owners, school deans and teachers, pupils and their parents, and local government authorities. Nevertheless, we were able to convince several counties to apply our solutions in practice over the past years.

In our joint talk we will address both the mathematical and the real-world aspects of this problem.

# Branch-and-Price for Combinatorial Slot Auctions

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When combinatorial bidding is permitted in auctions, such as FCC auction #31, the resulting winner-determination problem can become computationally challenging.

Here we want to report on joint work with O. Günlük and L. Ladányi (IBM Watson Research) indicating, that sometimes there is a very useful interaction between the bidding language and the winner-determination problem simplifying the latter. For FCC #31, Dietrich and Forrest (2002) propose a column generation formulation. Besides presenting results about our practical experience with this for auctions of spectrum slots we want to demonstrate the generality of this idea by showing how this can be applied to (airport-) slot-auctions, where start- and landing-rights at busy airport(-hubs) are to be allocated efficiently.

# Planning of buses and gates at Amsterdam Airport Schiphol

Guido Diepen

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All aircraft at Amsterdam Airport Schiphol (AMS) need to be assigned to a gate. These gates can be divided into two categories, ordinary gates and remote stands. Aircraft assigned to remote stands must be served by one or more buses to transport the passengers.

When looking at the gate assignment problem, one of the stages is the one-day-ahead planning during which a schedule for the next day is generated. The more robust this one-day-ahead schedule is, the less rescheduling will have to be done during the actual day when the real arrival and departure times deviate from the planned times.

The bus problem consists of planning for each bus which aircraft it must serve taking into account the legal constraints for the drivers. Here we again aim at finding a robust solution to avoid replanning during the day due to deviations to the arrival and departure times as much as possible.

For both of these problems of creating a schedule for the upcoming day, we present a solution approach based on column generation and for both problems we are able to find robust schedules in a matter of minutes.

We will also show how the proposed solution approach for the gate assignment problem can be implemented in AIMMS.



# An Integer Programming Approach to Robust Tail Assignment

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Due to the growth in air traffic and because of tight schedules, delays have become a major problem of airlines. Clearly, delayed flights and disrupted passenger connections increase operational costs and can harm the image of an airline. One way to deal with this problem is to add buffers to schedules. Such buffers are costly and should be used as effectively as possible. In addition to buffers, airlines use recovery actions such as aircraft swaps, flight cancellations, and ferry legs. Ideally, a flight plan would anticipate delays and maximize recovery opportunities at minimum cost; such a failure-proof schedule is called robust.

This talk considers a robust version of the tail assignment problem, which considers the routing of individual aircraft. We model this problem as an optimization problem in which non-robustness costs are minimized. Our approach is based on a stochastic model of the operation of an airline. In our talk we propose possible measures of robustness such as expected propagated delay, and solve the robust tail assignment problem by column generation technique.

# Annual Holiday Planning for the Crew of a Public Transport Company

Knut Haase

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We introduce a mathematical optimization approach for the determination of a annual holiday schedule for the crew of a public transport company. The objective is to maximize the preferences of differently qualified drivers taking into account that a sufficient number of drivers has to be available such that the transport service will be upheld each day. To do this, we have to know all the holiday applications for the next year. Moreover, for each application it can be defined an alternative, a time window in which the holiday should take place, and a minimum and a maximum length. Furthermore, we allow that two drivers can apply for common or non-common holidays, which is in particular important for couples which have school children. The problem formulation is related to the set covering problem and for its solution we present a column generation approach.

# Optimization Approaches in Airline and Railway Planning - A Comparative Survey

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Both areas, airline planning and railway planning, have attracted a significant interest from the operations research and optimization community during the past decades. Although both areas have significant similarities, i.e. transport of people and goods according to specified schedules, the dissimilarities seem to prevail due to the mostly separate developments of these fields.

In the presentation we focus on the main planning fields in railway and airline transport, discuss similarities and differences in these areas and the corresponding modeling and optimization approaches. In some cases, we see that similarities depend on the specific application instance, e.g. the size of a railway network or other aspects of the organizational framework.

# Routing Cars in Rail Freight Service

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Cars in rail freight services follow prescribed routes from their origin via intermediate shunting yards to their destination. The goal in designing such routes is to reduce the number of trains and their travel distances. Various real-world hard constraints make the problem difficult to formulate and also to solve. We present integer programming formulations for this car routing problem arising at the largest European railway company and discuss their pros and cons. One of our models uses exponentially many variables, and can thus computationally only be solved by column generation techniques. The models are compared on a set of real world instances.

# An overview of the use of branch-and-price in public transportation problems

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Public transportation has been a traditional area of application for integer programming approaches in general. Many problems have a multicommodity flow flavor which make them particularly amenable to branch-and-price approaches. We will have a look into the diverse applications in air and ground passenger transportation with a focus on how column generation and branch-and-price techniques are employed.

# A Pre-tactical Generalised Air Traffic Flow Management Problem

Karl Nachtigall

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We present a new model for solving a Generalised Air Traffic Flow Management Problem (GATFM), which combines a ground-holding problem with en-route air traffic flow management and, moreover, contains a runway assignment problem. This approach shall help to close the existing planning gap between the Europe-wide air pretactical traffic flow management process of Central Flow Management Unit (CFMU) of EUROCONTROL and the fine tuned, short term and airport focussed planning tools used in tactical planning. The main contributions of this paper is a novel, network based capacity management model. Based on this approach an integer linear programming problem formulation for the GATFM problem and a dedicated new solution approach, based on column generation, is deduced and discussed. The theoretical concept is generic and can be applied to a lot of similar problems in ATFM, especially to combine slot allocation and optimal re-routing. A prototyped version is implemented and evaluated by controllers. First results for real world data of the airport Frankfurt/ Main are presented and look very promising.

# Column Generation in Airline Planning and Control

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We give an overview of the *xOPT* optimization suite for solving crew and aircraft scheduling problems, developed at Lufthansa System Berlin. The algorithmic core of *xOPT* is a column generation framework combined with several reduction techniques, called reduce-and-generate approach. This technique makes it possible to compute almost globally optimal solutions of huge nonlinear scheduling problems in reasonable time. We discuss main challenges of this solution approach, and report numerical results on airline planning and control problems. References and further information can be found at:

<http://www.lhsystems.com/solutions/airline-solutions/airline-management-solutions/research-development/index.htm>

# Railway Crew Rescheduling

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The Dutch railway network experiences about three large disruptions per day on average. Disruption management tries to limit the effects of such disruptions for the passengers. The three main steps in the disruption management process are timetable update, rolling stock rescheduling and crew rescheduling.

In the first part of this talk we will present a novel algorithm for crew rescheduling. The algorithm is based on column generation techniques combined with Lagrangian heuristics. Since the number of duties is very large in practical instances, we embedded this algorithm into a dynamic duty selection schema.

In the second part of the talk we will present an extension of the basic crew rescheduling problem, namely crew rescheduling with retiming. This extension tries to give more flexibility to crew rescheduling by allowing delays for some trains. This simple integration of some timetabling decisions into crew rescheduling aims at improving the overall disruption management process. We show how the basic model can be extended and we will discuss how we adapted our column generation based algorithm. Computational results for real-life instances from Netherlands Railways, the largest passenger operator in the Netherlands, will be presented.



# Branch and Price Algorithm for Multimodal Evacuation Problems

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This article focuses on establishing efficient evacuation routes with bimodal transportation. We consider emergency management or event management situations such as football games, which assumes the absence of panic situations but still captures the several aspects of an evacuation settings. These include high demands during the event, need to satisfy demands quickly and congestion due to high demands. We assume private cars and buses as the modes of transportation. The cars are to take a path from source to destination, while the buses are routed. We assume that the routes of the buses are known. We need to establish efficient paths for the cars and determine the frequency of the buses along the routes. These problems are known to be NP-hard.

We present a Branch and Price algorithm to solve the multimodal evacuation problem to global optimality. Especially focus lies on several stabilization techniques implemented in order to enhance the computational efficiency. Some computational results are presented.

# Branch and Price for the Train Timetabling Problem

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The *train timetabling problem* (TTP) is to find, in a given railway network, a conflict-free set of train routes of maximum value. We present an integer programming formulation for this problem that is based on additional configuration variables to guarantee operational feasibility. Its LP-relaxation can be solved in polynomial time. These results are the theoretical basis for a column generation approach to tackle large-scale instances. Furthermore, we use the bundle method to solve a Lagrangean Relaxation instead of a LP Relaxation to speed up the algorithm. Computational results for the instances of the TTPLib 2008, a data library for train timetabling problems that can be accessed at <http://ttplib.zib.de>, are shown.

# Solution Techniques for Revenue Optimization

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In airline revenue management, recent models include the impact of customer choice behavior on demand forecasts and optimal control policies. The task is to find the best capacity allocation of number of seats to aircraft compartments and to find the best inventory control parameters - called *bid prices*.

In traditional airline revenue management customers buy a ticket in a certain booking class if this is available or do not buy at all otherwise. In reality, customers make use of having different booking options for selecting the best tickets for their planned trip from origin to destination. The criteria for a best ticket consist on the one hand of the ticket price and on the other hand of the perceived quality of connection, i.e. number of stops, travel time, and connection time.

Deriving the best possible control parameters for airline capacity control can be done by solving a huge dynamic program or a huge mixed-integer program. Both options seem to be infeasible due to curse of dimensionality of the state space or due to the number of possible control options and variables.

Recent solution approaches consists of column generation techniques or decomposition techniques. We give a short outlook on how column generation can be applied. Due to the strong computation time requirements of a few minutes, decomposition approaches seem to be more promising so that these are applied to real-world problems of thousands of flights per day.

We present a mathematical programming formulation for modeling the customer choice options and introduce a (deterministic) mixed integer program for solving the problem of optimizing the total network revenue for the given implied OD demand including adaptation of compartment capacities. Based on the result of this MIP, a decomposition into (flight) leg based problem takes places.

We discuss options for speeding up the solution of the mixed integer program for practical real-world applications. We conclude with the discussion of numerical results for real-world problems.