WIAS

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# Pricing of path-dependent cancelables

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Consider a path dependent cancelable contract which generates (possibly negative) cash-flows

$$C_1, \ldots, C_\tau$$

up to a cancelation date $\tau$. The cash-flows of this contract are equivalent to an aggregated cash-flow at cancellation date,

$$B_*(\mathcal{T}_\tau)\mathcal{Z}_\tau := B_*(\mathcal{T}_\tau)\sum_{j=1}^{\tau} Z_j,$$

with $Z_i := C_i/B_*(\mathcal{T}_i)$ being discounted cash-flows with respect to the numeraire $B_*$. Product price at time zero:

$$V_0^{cancel} := \sup_{\tau\in\{1,\ldots,k\}} E^{\mathcal{F}_0}\mathcal{Z}_\tau = \sup_{\tau\in\{1,\ldots,k\}} E^{\mathcal{F}_0}\sum_{j=1}^{\tau} Z_j,$$

where the supremum is taken over all stopping indices with values in the set $\{1, \ldots, k\}$.

## Path-dependent callables

A path dependent callable contract generates

$$C_{\tau+1}, \ldots, C_k$$

when called at $\tau$. It is equivalent to the sum of a non-callable and a cancelable one (and vice versa):

$$\begin{aligned} V_0^{call} &:= \sup_{\tau \in \{1,\ldots,k\}} E^0 \sum_{j=\tau+1}^{k} Z_j \\ &= E^0 \sum_{j=1}^{k} Z_j + \sup_{\tau \in \{1,\ldots,k\}} E^0 \sum_{j=1}^{\tau} (-Z_j) \end{aligned}$$

## Example: The cancelable snowball swap

Pays semi-annually a constant rate $I$ over the first year and in the forthcoming years $(\text{Previous coupon} + A - \text{Libor})^+$, semi-annually, where $A$ is given in the contract. For this case we take

$$
\begin{aligned}
K_i &:= I, \quad i = 0, 1, \\
K_i &:= (K_{i-1} + A_i - L_i(T_i))^+, \quad i = 2, ..., n-1,
\end{aligned}
$$

with $A_2 := 0.03$, $A_{i+1} = A_i$ for even $i$, $A_{i+1} = A_i + 0.005$ for odd $i$. Cancelation is allowed for $2 \leq \tau < n$, $n = 20$ (10 years)

Effective discounted cashflows at $T_j$:

$$
Z_j := \frac{(L_{j-1}(T_{j-1}) - K_{j-1})^+ \, \delta_{j-1}}{B_*(T_j)},
$$

hence aggregated up to cancelation $\mathcal{Z}_\tau = \sum_{j=1}^{\tau} Z_j$.

## Iterating the snowball swap

Take an input policy satisfying

$$i \leq \tau_i \leq k, \ \tau_k = k,$$
$$\tau_i > i \Rightarrow \tau_i = \tau_{i+1}, \qquad 0 \leq i < k,$$

construct the new policy

$$\begin{aligned} \widehat{\tau}_i &:= \inf\{j \geq i : \mathcal{Z}_j \geq \max_{p:\, j \leq p \leq k} E^{\mathcal{F}_j} \mathcal{Z}_{\tau_p}\} \\ &= \inf\{j \geq i : 0 \geq \max_{p:\, j \leq p \leq k} E^{\mathcal{F}_j} \sum_{q=j+1}^{\tau_p} Z_q\} \end{aligned}$$

and compute the iterated price

$$\widehat{Y}_0 := E^{\mathcal{F}_0} \mathcal{Z}_{\widehat{\tau}_0},$$

which is generally an improvement of $Y_0$ due to policy $\tau$.

## Numerical results for typical market data

### Improved Andersen

| $d$ | $Y(0;\tau_A)$ (SD) | $\widehat{Y}(0;\tau_A)$ (SD) | $Y_{up}(0;\tau_A)$ (SD) |
|---|---|---|---|
| 1 | 127.77(0.238) | 129.77(0.318) | 130.33(0.247) |
| 2 | 114.93(0.231) | 120.00(0.389) | 121.92(0.293) |
| 19 | 76.725(0.217) | 91.600(0.460) | 98.107(0.476) |

150 000 outer and 500 inner paths for $\widehat{Y}$ and 20 000 outer (with 500 inner) paths for $Y_{up}$.

### Improved least-squares regression method (Piterbarg)

| $d$ | $Y(0;\tau_{LS})$ (SD) | $\widehat{Y}(0;\tau_{LS})$ (SD) | $Y_{up}(0;\tau_{LS})$ (SD) |
|---|---|---|---|
| 1 | 117.73(0.243) | 128.81(0.632) | 132.28(0.313) |
| 2 | 103.70(0.238) | 120.73(0.466) | 123.54(0.346) |
| 19 | 74.913(0.224) | 93.515(0.469) | 97.479(0.379) |

200 000 outer and 500 inner paths for $\widehat{Y}$ and 20 000 outer (with 500 inner) paths for $Y_{up}$.

## Improving an Andersen-like optimization of the LS exercise boundary

| $d$ | $Y(0;\tau_{LS\text{-}A})$ (SD) | $\widehat{Y}(0;\tau_{LS\text{-}A})$ (SD) | $Y_{up}(0;\tau_{LS\text{-}A})$ (SD) |
|---|---|---|---|
| 1 | 129.58(0.237) | 128.70(0.349) | 130.24(0.244) |
| 2 | 119.58(0.230) | 118.95(0.345) | 120.77(0.244) |
| 10 | 92.201(0.219) | 97.376(0.456) | 100.20(0.418) |
| 19 | 87.787(0.217) | 94.487(0.445) | 95.843(0.430) |

150 000 outer and 100 inner paths for $\widehat{Y}$ and 5000 outer (with 500 inner) paths for $Y_{up}$.

$$\tau_{LS\text{-}A,\ i} = \inf\{j \geq i : \mathcal{Z}_j \geq H_j + Y_{LS,\ j}\}$$

with optimized constants $H_j$.

Message:

(i) Price the callable using Pitterbarg's version of Longstaff-Schwartz;

(ii) Improve the obtained exercise boundary with an Andersen-like optimization;

(iii) Compute the Dual upperbound due to the stopping time $\tau_{LS\text{-}A,\ 0}$;

(iv) If there is still a significant gap between lower and upper bound, then improve the policy $\tau_{LS\text{-}A,\ i}$ by the iteration method.

Bender, Kolodko, Schoenmakers (2006), revised version of WIAS prepr. 1061