Applications of Stochastic Programming in Electricity Portfolio and Airline Revenue Management

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Introduction

Practical optimization models often contain uncertain parameters or stochastic processes. In many cases it is not appropriate to replace the uncertain parameters by their mean values or some other statistical estimate. Alternatives are robust/worst case optimization models or, if statistical data is available, modelling the relevant stochastic process by a finite number of scenarios with given probabilities and incorporating them into the optimization model. This leads to stochastic optimization models having the advantages:

- Solutions are robust with respect to uncertain changes of the data.
- The risk of decisions can be measured, controlled or (even) minimized.
- Recent simulation studies show that "stochastic solutions" are advantageous compared to deterministic ones (e.g. Schmöller 04 at RWTH Aachen).

The presentation will focus on

- Stochastic optimization models for electricity portfolio and airline revenue management and their solution by Lagrangian relaxation.
- Modelling of scenario trees for the relevant stochastic processes.
- Measuring and minimizing risk, in particular, in electricity portfolio management models.

Electricity Portfolio Management

schedule

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schedule

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electric power company

bidding

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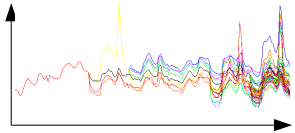
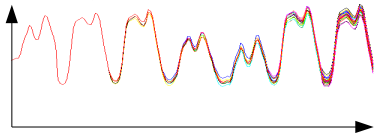
EEX

power exchange

electricity spot prices



electricity demand



We consider the electricity portfolio management of an electric power company. Its portfolio consists of the following positions:

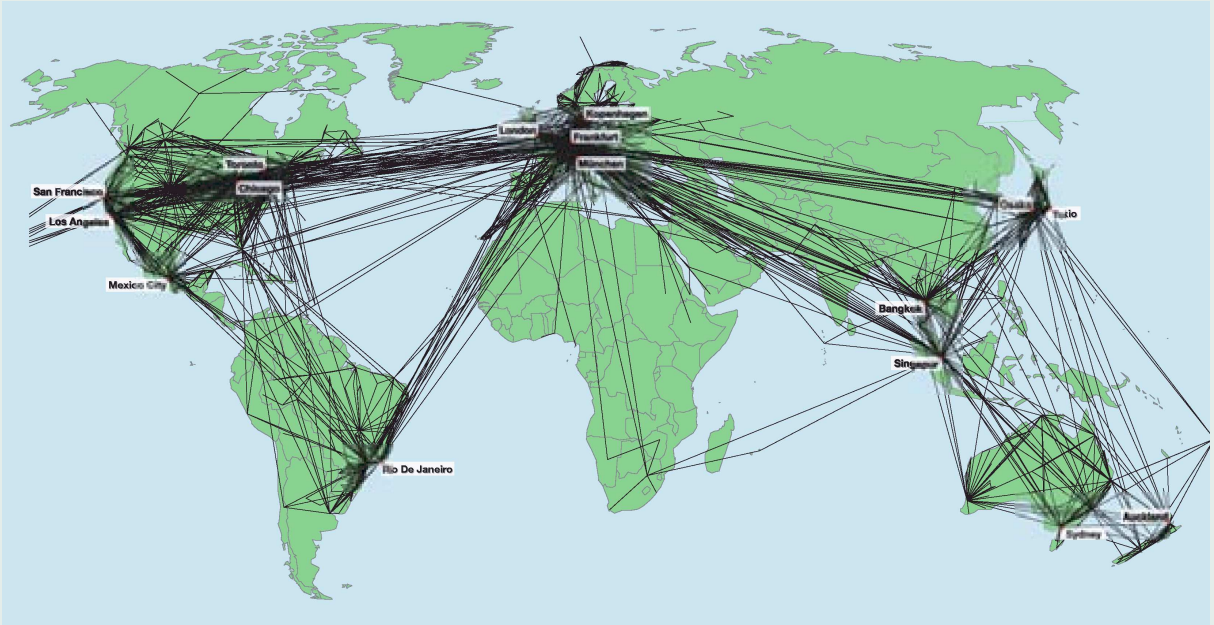
- power production (based on company-owned thermal units),
- bilateral contracts,
- (physical) (day-ahead) spot market trading (e.g., EEX) and
- (financial) trading of futures.

The time horizon is discretized into hourly intervals. The underlying stochasticity consists in a multivariate stochastic load and price process that is approximately represented by a finite number of scenarios. The objective is to maximize the total expected revenue. The portfolio management model is a large scale mixed-integer multistage stochastic program.

Should the expected revenue be maximized exclusively or should the risk of its production and trading decisions simultaneously be bounded or even minimized ?

Airline Revenue Management

Origin&Destination (O&D) Revenue Management has become standard in airline industry. It considers the entire airline network. Earlier optimization models make assumptions on the passenger demand process and incorporate simplifications.

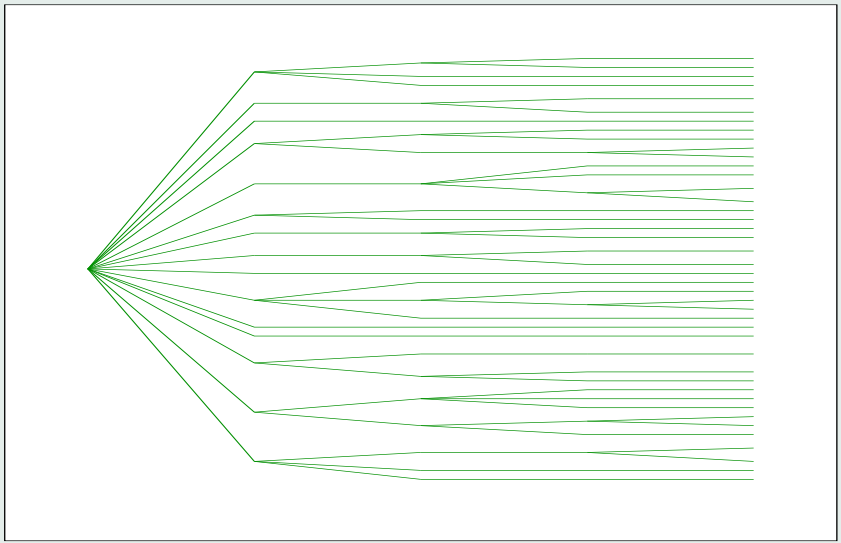


Our optimization model for O&D Revenue Management has the following features: It

- incorporates the stochastic nature of the passenger behaviour,
- determines protection levels for all origin destination itineraries, fare classes, points of sale and data collection points (dcp's),
- models the dynamic booking control process consisting of recursive decisions and observations,
- represents a multi-stage stochastic program where its stages correspond to the dcp's of the booking horizon,
- reduces to a specially structured large scale MIP if the stochastic passenger demand process is represented by a finite number of scenarios with probabilities.

Uncertainty:
Electricity or heat demands, market prices, inflows / passenger behavior over time (represented by a variety of scenarios with their probabilities)

Modeling requirement:
Process of recursive observations and decisions (represented by possible branchings of scenarios at prescribed time points)



Tree of scenarios for the future

Mathematical Model

Let $\{\xi_t\}_{t=1}^T$ be a discrete-time stochastic data process defined on some probability space $(\Omega, \mathcal{F}, I\!P)$ and with ξ_1 deterministic. The stochastic decision x_t at period t is assumed to be measurable with respect to $\mathcal{F}_t = \mathcal{F}_t(\xi) := \sigma(\xi_1, \ldots, \xi_t)$ (nonanticipativity).

Multistage stochastic optimization model:

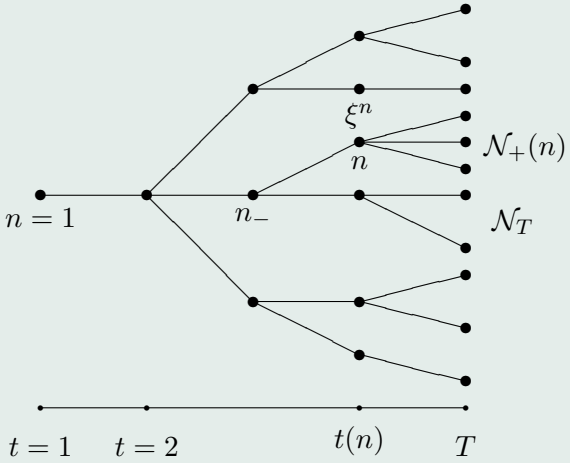
$$\min \left\{ I\!E \left[\sum_{t=1}^{T} \langle b_t(\xi_t), x_t \rangle \right] \middle| \begin{array}{l} x_t \in X_t, t = 1, \ldots, T, A_{1,0} x_1 = h_1(\xi_1), \\ x_t \text{ is } \mathcal{F}_t - \text{measurable}, t = 1, \ldots, T, \\ A_{t,0} x_t + A_{t,1} x_{t-1} = h_t(\xi_t), t = 2, ., T \end{array} \right\}$$

where the sets X_t, $t = 1, \ldots, T$, are polyhedral cones, the vectors $b_t(\cdot)$ and $h_t(\cdot)$ depend affine linearly on ξ_t.

If the process $\{\xi_t\}_{t=1}^T$ has a finite number of scenarios, they exhibit a scenario tree structure.

Data process approximation by scenario trees

The process $\{\xi_t\}_{t=1}^T$ is approximated by a process forming a scenario tree being based on a finite set $\mathcal{N} \subset I\!N$ of nodes.



Scenario tree with $T = 5$, $N = 22$ and 11 leaves

$n = 1$ root node, n_- unique predecessor of node n, $\mathrm{path}(n) = \{1, \ldots, n_-, n\}$, $t(n) := |\mathrm{path}(n)|$, $\mathcal{N}_+(n)$ set of successors to n, $\mathcal{N}_T := \{n \in \mathcal{N} : \mathcal{N}_+(n) = \emptyset\}$ set of leaves, $\mathrm{path}(n)$, $n \in \mathcal{N}_T$, scenario with (given) probability π^n, $\pi^n := \sum_{\nu \in \mathcal{N}_+(n)} \pi^\nu$ probability of node n, ξ^n realization of $\xi_{t(n)}$.

Tree representation of the optimization model

$$\min\left\{\sum_{n\in\mathcal{N}}\pi^n\langle b_{t(n)}(\xi^n),x^n\rangle \,\middle|\, \begin{array}{l} x^n\in X_{t(n)}, n\in\mathcal{N}, A_{1,0}x^1=h_1(\xi^1) \\ A_{t(n),0}x^n+A_{t(n),1}x^{n-}=h_{t(n)}(\xi^n), n\in\mathcal{N}\end{array}\right\}$$

How to solve the optimization model ?

- Standard software (e.g., CPLEX)

- Decomposition methods for (very) large scale models

  (Ruszczynski/Shapiro (Eds.): Stochastic Programming, Handbook, 2003)

Open questions:

- Which decomposition scheme should be used ?

- How to generate (multivariate) scenario trees ?

- How to model and incorporate risk ?

Geographical decomposition

Often the tree representation of the multistage stochastic program has block separable structure

$$\min\left\{\sum_{n\in\mathcal{N}}\pi^n\sum_{i=1}^{k}\langle b^i_{t(n)}(\xi^n),x^n_i\rangle \;\middle|\; \begin{array}{l} x^n_i\in X^i_{t(n)},\ A^i_{1,0}x^1_i=h^i_1(\xi^1)\\ \sum_{i=1}^k B^i_{t(n)}(\xi^n)x^n_i\geq g_{t(n)}(\xi^n)\\ A^i_{t(n),0}x^n_i+A^i_{t(n),1}x^{n-}_i=h^i_{t(n)}(\xi^n)\\ i=1,\dots,k,\ n\in\mathcal{N}\end{array}\right\}$$

Lagrange relaxation of coupling constraints:

$L(x,\lambda)=$

$$\sum_{n\in\mathcal{N}}\pi^n\Big(\sum_{i=1}^{k}\langle b^i_{t(n)}(\xi^n),x^n_i\rangle+\langle\lambda^n,(g_{t(n)}(\xi^n)-\sum_{i=1}^{k}B^i_{t(n)}(\xi^n)x^n_i)\rangle\Big)$$

The dual problem

$$\max_{\lambda\geq 0}\ \inf_x L(x,\lambda)$$

decomposes into k geograhical subproblems and is solved by bundle subgradient methods followed by a Lagrangian heuristics.

Electricity portfolio management

Stochastic process: $\{\xi_t = (d_t, \gamma_t, \alpha_t, \beta_t, \zeta_t)\}_{t=1}^T$
(electrical load, inflows, (fuel or electricity) prices) given as a (multivariate) scenario tree.

Mixed-integer programming problem:

$$\min \sum_{n\in\mathcal{N}} \pi^n \sum_{i=1}^{I} [C_i^n(p_i^n, u_i^n) + S_i^n(u_i)] \quad \text{s.t.}$$

$$p_{it(n)}^{\min} u_i^n \le p_i^n \le p_{it(n)}^{\max} u_i^n, \quad u_i^n \in \{0,1\}, \quad n \in \mathcal{N},\ i = 1,\dots,I,$$

$$u_i^{n-\tau} - u_i^{n-(\tau+1)} \le u_i^n, \quad \tau = 1,\dots,\bar{\tau}_i - 1,\ n \in \mathcal{N},\ i = 1,\dots,I,$$

$$u_i^{n-(\tau+1)} - u_i^{n-\tau} \le 1 - u_i^n, \quad \tau = 1,\dots,\underline{\tau}_i - 1,\ n \in \mathcal{N},\ i = 1,\dots,I,$$

$$0 \le v_j^n \le v_{jt(n)}^{\max},\ 0 \le w_j^n \le w_{jt(n)}^{\max},\ 0 \le l_j^n \le l_{jt(n)}^{\max},\ n \in \mathcal{N},\ j = 1,\dots,J,$$

$$l_j^n = l_j^{n-} - v_j^n + \eta_j w_j^n + \gamma_j^n, \quad n \in \mathcal{N},\ j = 1,\dots,J,$$

$$l_j^0 = l_j^{\text{in}}, \quad l_j^n = l_j^{\text{end}}, \quad n \in \mathcal{N}_T,\ j = 1,\dots,J,$$

$$\sum_{i=1}^{I} p_i^n + \sum_{j=1}^{J} (v_j^n - w_j^n) \ge d^n, \quad n \in \mathcal{N},$$

$$\sum_{i=1}^{I} (u_i^n p_{it(n)}^{\max} - p_i^n) \ge r^n, \quad n \in \mathcal{N}.$$

Here C_i^n are fuel or trading costs and S_i^n start-up costs of unit i at node $n \in \mathcal{N}$:

$$C_i^n(p_i^n, u_i^n) := \max_{l=1,\dots,\bar{l}} \{\, \alpha_{il}^n p_i^n + \beta_{il}^n u_i^n \,\} \qquad S_i^n(u_i) := \max_{\tau=0,\dots,\tau_i^c} \zeta_{i\tau}^n (u_i^n - \sum_{\kappa=1}^{\tau} u_i^{n-\kappa})$$

Stochastic O&D airline revenue management

Objective

$$\max_{(P^n_{i,j,k})} \left\{ \sum_{n=1}^{N} \pi^n \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \left[f^b_{i,j,k,t(n)} b^n_{i,j,k} - f^c_{i,j,k,t(n)} c^n_{i,j,k} \right] \right\}$$

Constraints

Cumulative bookings

$$B^s_{i,j,k,0} := \bar{B}^0_{i,j,k}; \qquad B^n_{i,j,k} := B^{n-}_{i,j,k} + b^n_{i,j,k}$$

Cumulative cancelations ($\vartheta \in (0.0, 0.5]$)

$$\gamma^n_{i,j,k} B^n_{i,j,k} - \vartheta \leq C^n_{i,j,k} < \gamma^n_{i,j,k} B^n_{i,j,k} + \vartheta$$

Cancelations

$$c^n_{i,j,k} = C^n_{i,j,k} - C^{n-}_{i,j,k}$$

Passenger demands

$$b^n_{i,j,k} + z^{b,n}_{i,j,k} = d^n_{i,j,k}$$

Protection levels

$$B^n_{i,j,k} - C^n_{i,j,k} + z^{P,n}_{i,j,k} = P^{n-}_{i,j,k}$$

Number of bookings (disjunctive constraints) ($\omega > 0$, adequately large)

$$0 \leq z^{b,n}_{i,j,k} \leq (1 - \tilde{z}^n_{i,j,k}) d^n_{i,j,k} \quad 0 \leq z^{P,n}_{i,j,k} \leq \tilde{z}^n_{i,j,k} \omega \quad \tilde{z}^n_{i,j,k} \in \{0, 1\}$$

Leg capacity limits

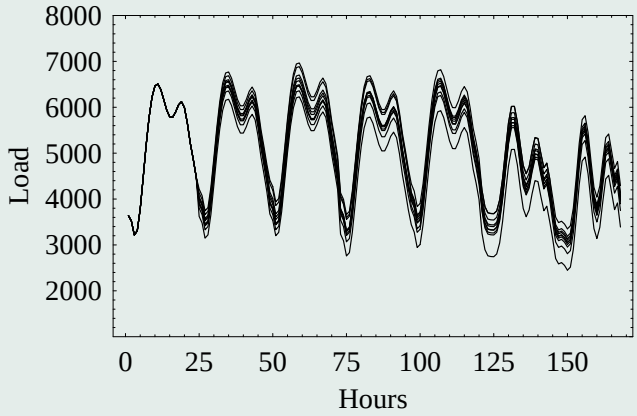
$$\sum_{i \in \mathcal{I}_l} \sum_{j \in \mathcal{J}_m(l)} \sum_{k=1}^{K} P^n_{i,j,k} \leq C_{l,m} \qquad \forall n \in \mathcal{N}_{T-1}$$

Integrality and nonnegativity constraints

$$B^n_{i,j,k}, C^n_{i,j,k}, P^n_{i,j,k} \in \mathbb{Z}; \qquad b^n_{i,j,k} \geq 0; \qquad c^n_{i,j,k} \geq 0$$

Generation of scenario trees

(i) Development of a stochastic model for the data process ξ (parametric [e.g. time series model], nonparametric [e.g. resampling])



Scenarios of a weekly electrical load process

and generation of simulation scenarios;

(ii) Construction of a scenario tree out of the stochastic model or of the simulation scenarios.

Approaches for (ii):

(1) Bound-based approximation methods,
(Frauendorfer 96, Kuhn 05, Edirisinghe 99, Casey/Sen 05).

(2) Monte Carlo-based schemes (inside or outside decomposition methods) (e.g. Shapiro 03, 06, Higle/Rayco/Sen 01, Chiralaksanakul/Morton 04).

(3) the use of Quasi Monte Carlo integration quadratures
(Pennanen 05, 06).

(4) EVPI-based sampling schemes (inside decomposition schemes)
(Corvera Poire 95, Dempster 04).

(5) Moment-matching principle (Høyland/Wallace 01, Høyland/Kaut/Wallace 03).

(6) (Nearly) best approximations based on probability metrics
(Pflug 01, Hochreiter/Pflug 02, Gröwe-Kuska/Heitsch/Römisch 01, 03, Heitsch/Römisch 05).

Survey: Dupačová/Consigli/Wallace 2000

Stability of multistage models

Stability studies the behavior of the optimization problem if the input ξ is approximated.

We assume that the stochastic input process $\xi = \{\xi_t\}_{t=1}^T$ belongs to the linear space $\times_{t=1}^T L_r(\Omega, \mathcal{F}, I\!P; I\!R^d)$ for some $r > 1$. The optimization model is regarded in the space $\times_{t=1}^T L_{r'}(\Omega, \mathcal{F}, I\!P; I\!R^{m_t})$ for some $r' > 1$, where both spaces are endowed with the norms

$$\|\xi\|_r := \Big(\sum_{t=1}^T I\!E[|\xi_t|^r]\Big)^{\frac{1}{r}}$$

$$\|x\|_{r'} := \Big(\sum_{t=1}^T I\!E[|x_t|^{r'}]\Big)^{\frac{1}{r'}}$$

respectively. Here, $|\cdot|$ denotes some norm on the relevant Euclidean spaces and r' is defined by

$$r' := \begin{cases} \frac{r}{r-1} & , \text{ if costs are random ,} \\ r & , \text{ if only right-hand sides are random.} \end{cases}$$

Let $v(\xi)$ denote the optimal value of the stochastic optimization model.

Assumptions:

(A1) $\mathbb{E}[|\xi|^r] < \infty$,

(A2) The optimization model has relatively complete recourse,

(A3) The objective function is level-bounded locally uniformly at ξ.

Theorem: (Heitsch/Römisch/Strugarek 05)

Let (A1), (A2) and (A3) be satisfied.

There exist constants $L > 0$ and $\delta > 0$ such that the estimate

$$|v(\xi) - v(\tilde{\xi})| \leq L(\|\xi - \tilde{\xi}\|_r + D_{\mathrm{f}}(\xi, \tilde{\xi}))$$

holds for all $\tilde{\xi} \in L_r(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{R}^s)$ with $\|\tilde{\xi} - \xi\|_r \leq \delta$.

Here, D_{f} is the filtration distance

$$D_{\mathrm{f}}(\xi, \tilde{\xi}) = \inf \sum_{t=2}^{T-1} \max\{\|x_t - \mathbb{E}[x_t|\mathcal{F}_t(\tilde{\xi})]\|_{r'}, \|\tilde{x}_t - \mathbb{E}[\tilde{x}_t|\mathcal{F}_t(\xi)]\|_{r'}\},$$

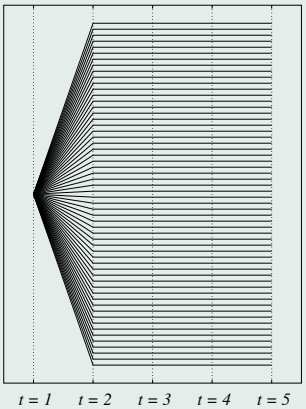
where x and $\tilde{x}$ are solutions of the stochastic optimization problem.

Constructing scenario trees

Let ξ be the original stochastic process on some probability space $(\Omega, \mathcal{F}, I\!P)$ with parameter set $\{1, \ldots, T\}$ and state space $I\!R^d$. We aim at generating a scenario tree $\xi^{\rm tr}$ such that

$$\|\xi - \xi^{\rm tr}\|_r \qquad \text{and} \qquad D_{\rm f}(\xi, \xi^{\rm tr})$$

are small and, hence, the optimal values $v(\xi)$ and $v(\xi^{\rm tr})$ are close to each other. Since this problem is hardly solvable in general, we replace ξ by a finitely discrete approximation $\xi^{\rm f}$ such that $\|\xi - \xi^{\rm f}\|_r$ is small and its scenarios $\xi^i = (\xi^i_1, \ldots, \xi^i_T)$ with probabilities π^i, $i = 1, \ldots, N$ form a *fan* of individual scenarios.



An **algorithm** was developed that generates a tree ξ^{tr} by deleting and bundling scenarios at each $t = 2, \ldots, T$ (that are close to each other) and such that

$$\|\xi^{\mathrm{f}} - \xi^{\mathrm{tr}}\|_r$$

may be computed and bounded and that

$$D_{\mathrm{f}}(\xi^{\mathrm{f}}, \xi^{\mathrm{tr}})$$

may be bounded from above. The latter relies on the

Proposition:
Let (A2) and (A3) be satisfied. Then there exists a constant $\hat{L} > 0$ such that the filtration distance allows the estimate

$$D_{\mathrm{f}}(\xi^{\mathrm{f}}, \xi^{\mathrm{tr}}) \leq \hat{L} \begin{cases} \left(\sum_{i \in I_2} \sum_{j \in I_{2,i}} p_j \|\xi^j - \xi^i\|^{r'} \right)^{\frac{1}{r'}} & , 1 \leq r' < \infty \\ \max_{i \in I_2} \max_{j \in I_{2,i}} \|\xi^j - \xi^i\| & , r' = \infty \,. \end{cases}$$

Tolerances ε_r and ε_{f} are prescribed for $\|\xi^{\mathrm{f}} - \xi^{\mathrm{tr}}\|_r$ and $D_{\mathrm{f}}(\xi^{\mathrm{f}}, \xi^{\mathrm{tr}})$, respectively, which control the scenario tree generation process.

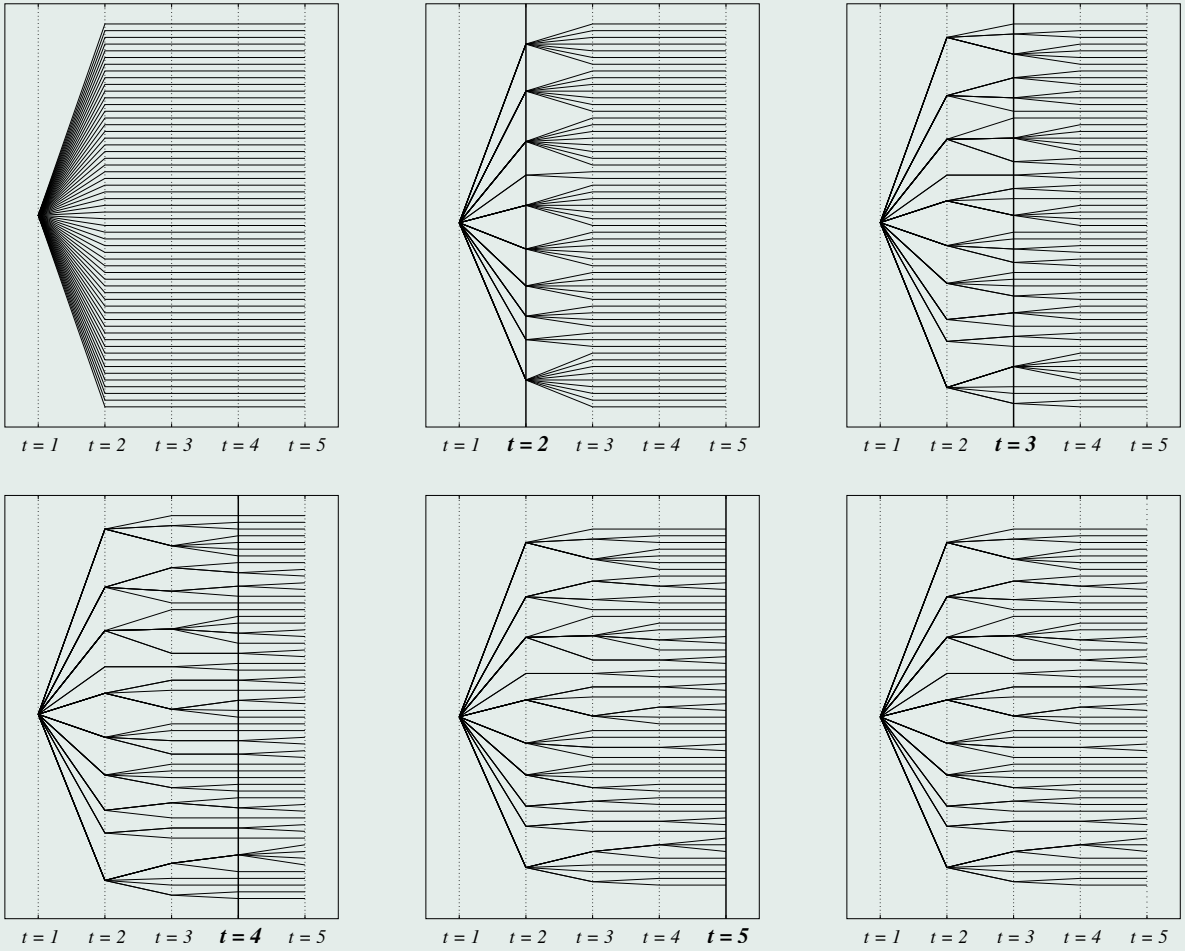


Illustration of the forward tree construction for an example including T=5 time periods starting with a scenario fan containing N=58 scenarios

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Risk functionals

Let z be a real random variable on some probability space $(\Omega, \mathcal{F}, I\!P)$. Assume that $z = z(x)$ is the revenue depending on a decision x in some stochastic optimization model. The traditional objective of such models consists in maximizing the expected revenue, i.e.,

$$\max_x \; I\!E[z(x)].$$

However, the revenue $z(x)$ of some or many decisions x might have fat tails, in particular, to the left. Looking only at the expectation of $z(x)$ hides any tail information.

Examples of risk functionals:

Upper semivariance:

$$sV_+(z) := I\!E[[I\!E[z] - z]_+^2] = I\!E[\max\{I\!E[z] - z, 0\}^2]$$

Value-at-Risk:

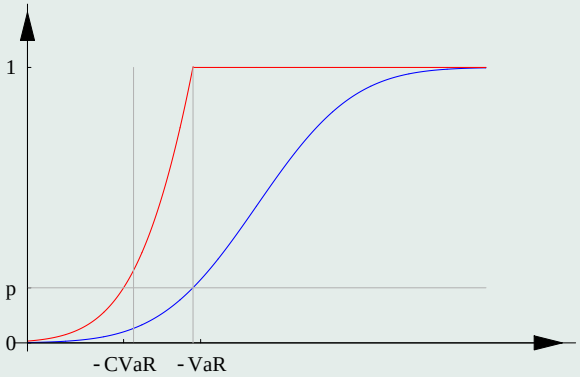
$$VaR_p(z) := -\inf\{r \in I\!R : I\!P(z \le r) \ge p\} \quad (p \in (0,1))$$

Conditional Value-at-risk:

$CVaR_p(z) :=$ mean of the tail distribution function F_p

where $F_p(t) := \begin{cases} 1 & t \geq -VaR_p(z), \\ \frac{F(t)}{p} & t < -VaR_p(z) \end{cases}$ and

$F(t) := I\!P(\{z \leq t\})$ is the distribution function of z.



$VaR_p(z)$ and $CVaR_p(z)$ for a continuously distributed z

Axiomatic characterization of risk:

Let $\mathcal{Z}$ denote a linear space of real random variables on some probability space $(\Omega, \mathcal{F}, I\!P)$. We assume that $\mathcal{Z}$ contains the constants. A functional $\rho : \mathcal{Z} \to I\!R$ is called a risk measure if it satisfies the following two conditions for all $z, \tilde{z} \in \mathcal{Z}$:

(i) If $z \leq \tilde{z}$, then $\rho(z) \geq \rho(\tilde{z})$ (monotonicity).

(ii) For each $r \in I\!R$ we have $\rho(z + r) = \rho(z) - r$ (equivariance).

A risk measure ρ is called convex if it satisfies the condition

$$\rho(\lambda z + (1 - \lambda)\tilde{z}) \leq \lambda\rho(z) + (1 - \lambda)\rho(\tilde{z})$$

for all $z, \tilde{z} \in \mathcal{Z}$ and $\lambda \in [0, 1]$.
A convex risk measure is called coherent if it is positively homogeneous, i.e., $\rho(\lambda z) = \lambda\rho(z)$ for all $\lambda \geq 0$ and $z \in \mathcal{Z}$.

References: Artzner/Delbaen/Eber/Heath 99, Föllmer/Schied 02

Multiperiod polyhedral risk functionals

When a stochastic process $z = \{z_t\}_{t=1}^T$ with $z_t \in L_p(\Omega, \mathcal{F}_t, I\!P)$, $1 \leq p \leq +\infty$, is considered that evolves over time and unveils the available information with the passing of time, it may become necessary to use multiperiod risk functionals. Then we need to consider the filtration of σ-fields adapted to z, i.e., $\mathcal{F}_t = \sigma\{z_1, \ldots, z_t\}$, $t = 1, ..., T$. Then $\mathcal{F}_t \subseteq \mathcal{F}_{t+1} \subseteq \mathcal{F}$, and $\mathcal{F}_1 = \{\emptyset, \Omega\}$, i.e. that z_1 is always deterministic.

Definition: (Artzner et al. 01, 02)
A functional ρ on $\times_{t=1}^T L_p(\Omega, \mathcal{F}_t, I\!P)$ is called multiperiod risk functional if the conditions
(i) If $z \leq \tilde{z}$ a.s., then $\rho(z) \geq \rho(\tilde{z})$ *(monotonicity)*,
(ii) For each $r \in I\!R$ we have $\rho(z + r) = \rho(z) - r$ *(equivariance)*,
are satisfied. It is called a multiperiod convex (coherent) risk functional, if ρ is convex (and positively homogeneous) in addition.

It is a natural idea to introduce risk functionals as optimal values of certain multistage stochastic programs.

Definition: A multiperiod risk functional ρ on $\times_{t=1}^{T} L_p(\Omega, \mathcal{F}_t, I\!P)$ is called polyhedral if there are $k_t \in I\!N$, $c_t \in I\!R^{k_t}$, $t = 1, \ldots, T$, $w_{t\tau} \in I\!R^{k_{t-\tau}}$, $t = 1, \ldots, T$, $\tau = 0, ..., t-1$, and polyhedral cones $Y_t \subset I\!R^{k_t}$, $t = 1, \ldots, T$, such that

$$\rho(z) = \inf \left\{ I\!E \left[\sum_{t=1}^{T} \langle c_t, y_t \rangle \right] \middle| \begin{array}{c} y_t \in L_p(\Omega, \mathcal{F}_t, I\!P; I\!R^{k_t}),\ y_t \in Y_t \\ \sum_{\tau=0}^{t-1} \langle w_{t,\tau}, y_{t-\tau} \rangle = z_t,\ \ t = 1, \ldots, T \end{array} \right\}.$$

Remark: A convex combination of (negative) expectation and of a multiperiod polyhedral risk functional is again a multiperiod polyhedral risk functional.

Polyhedral risk functionals preserve linearity and decomposition structures of optimization models.

(Eichhorn/Römisch: SIAM J. Opt. 05)

Theory: Characterization of coherent multiperiod polyhedral risk functionals based on duality results.

Example: (Multiperiod extensions of CVaR)
A first idea is to incorporate the Conditional-Value-at-Risk at all time periods and to consider the weighted sum

$$\rho_1(z) := \sum_{t=2}^{T} \gamma_t CVaR_{\alpha_t}(z_t) = \sum_{t=2}^{T} \gamma_t \inf_{r \in I\!R} \left\{ r + \frac{1}{\alpha_t} I\!E \left[(r+z)^- \right] \right\}$$

with some weights $\gamma_t \geq 0$, $\sum_{t=1}^{T} \gamma_t = 1$, and some confidence levels $\alpha_2, \alpha_3, ..., \alpha_T \in (0,1)$. Here, $a^- = \max\{0, -a\}$.
Then ρ_1 is a coherent polyhedral multiperiod risk functional.
By interchanging sum and minimization one arrives at the variant

$$\rho_2(z) = \inf_{r \in I\!R} \left\{ r + \sum_{t=2}^{T} \beta_t I\!E \left[(z_t + r)^- \right] \right\}.$$

It is also coherent and polyhedral.

Electricity portfolio management: statistical models and scenario trees

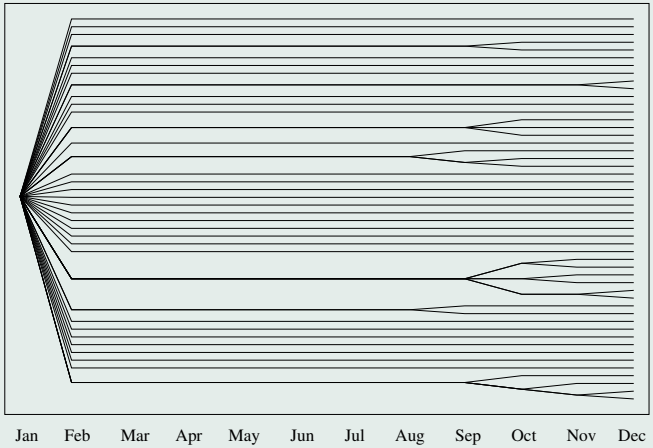
For the stochastic input data of the optimization model here (yearly electricity and heat demand, and electricity spot prices), a statistical model is employed. It is adapted to historical data in the following way:

- cluster classification for the intra-day (demand and price) profiles

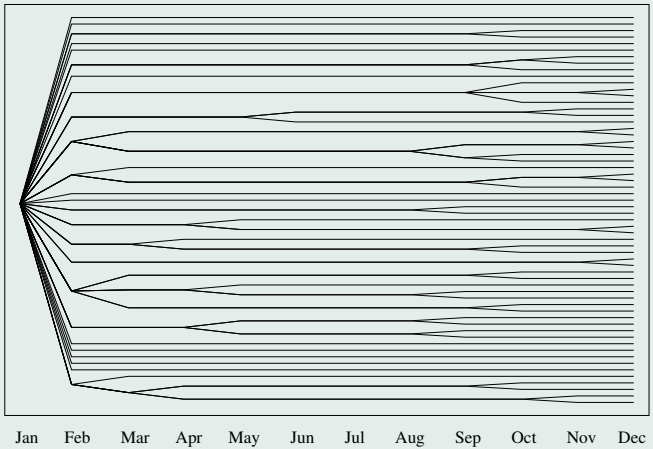
- 3-dimensional time series model for the daily average values (deterministic trend functions, a trivariate ARMA model for the (stationary) residual time series)

- simulation of an arbitrary number of three dimensional sample paths (scenarios) by sampling the white noise processes for the ARMA model and by adding on the trend functions and matched intra-day profiles from the clusters afterwards.

- generation of scenario trees by the forward tree construction technique



a) Modified forward tree construction with filtration level 0.6



b) Modified forward tree construction with filtration level 0.7

Yearly demand-price scenario trees with relative reduction level 0.5 (Heitsch/Römisch 05)

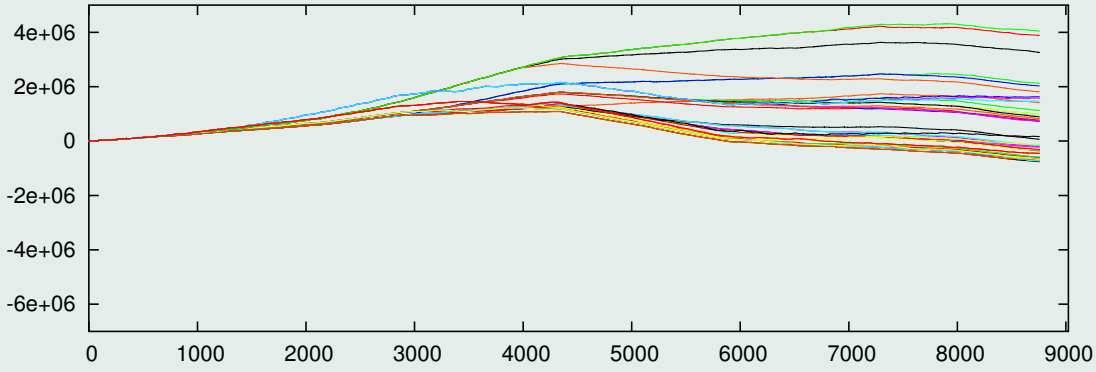
Electricity portfolio management: Results

Test runs were performed on real-life data of the utility DREWAG Stadtwerke Dresden GmbH leading to a linear programs containing $T = 365 \cdot 24 = 8760$ time steps, a scenario tree with 40 demand-price scenarios and about $N = 150.000$ nodes. The objective function is of the form

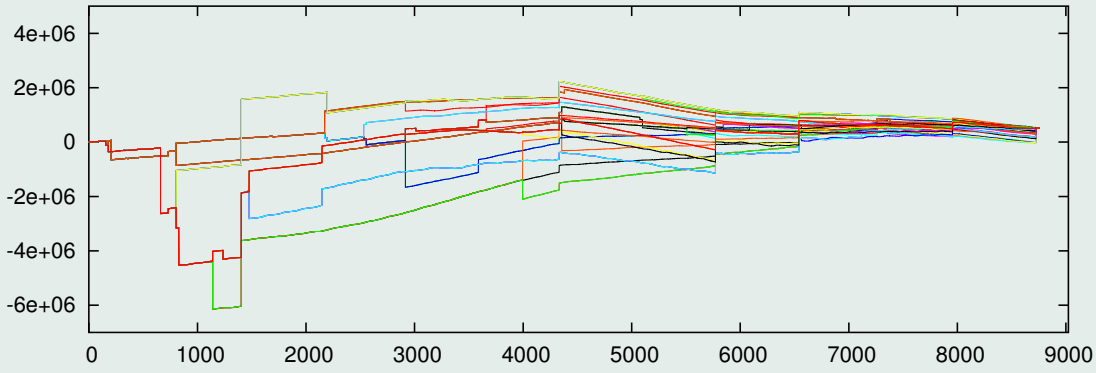
$$\text{Minimize} \quad \gamma\rho(z) - (1-\gamma)\mathbb{E}[z_T]$$

with a (multiperiod) risk functional ρ with coefficient $\gamma \in [0,1]$ ($\gamma = 0$ corresponds to no risk). $\mathbb{E}[z_T]$ denotes the overall expected revenue.

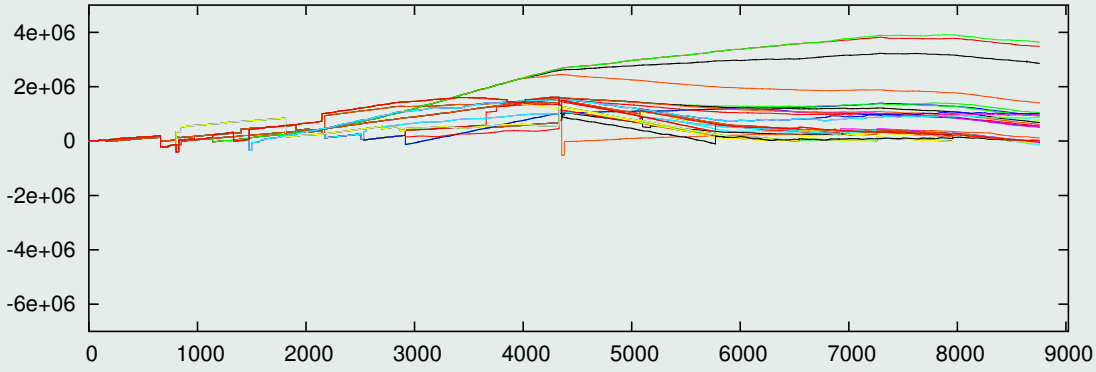
The model is implemented and solved with ILOG CPLEX 9.1 on a 2 GHz Linux PC with 1 GB memory.



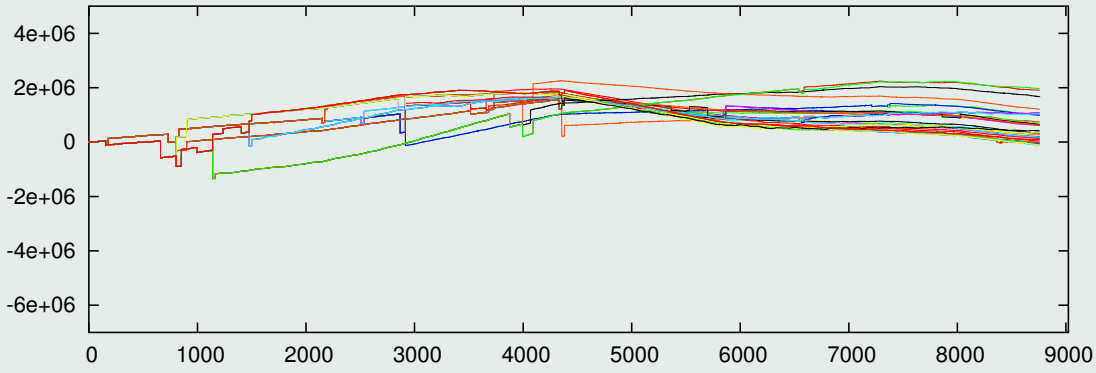
Total revenue and $\gamma = 0$



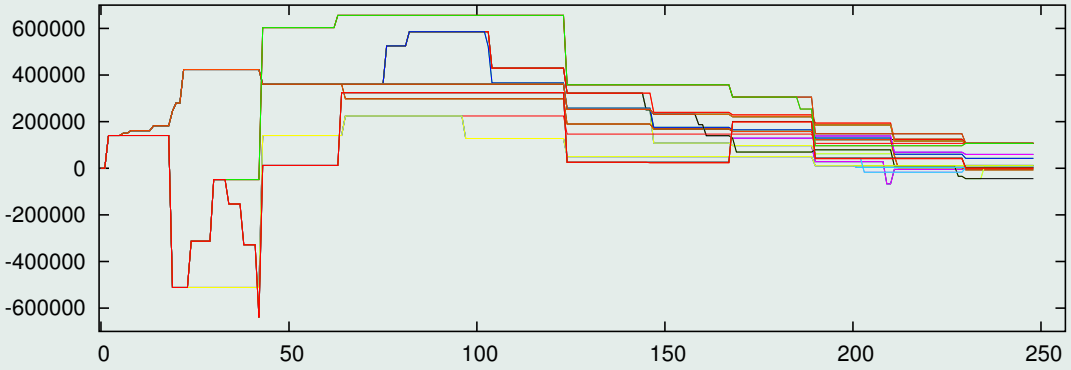
Total revenue with $CVaR_{0.05}$ and $\gamma = 0.9$



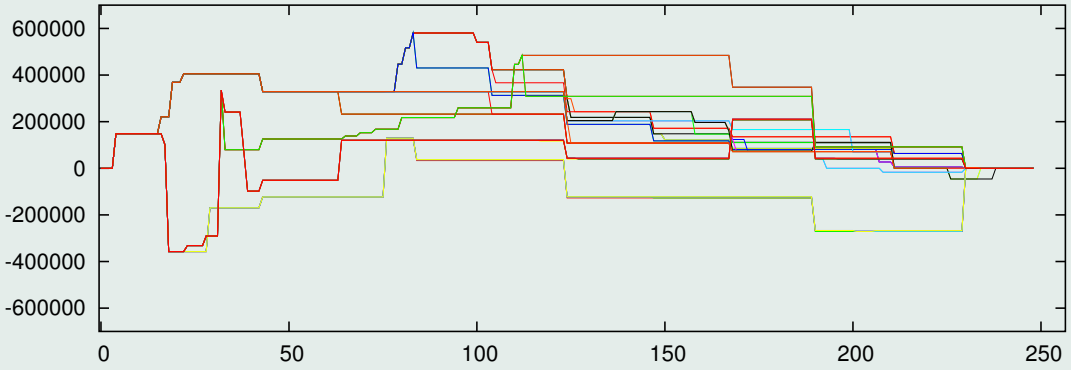
Total revenue with ρ_2 and $\gamma = 0.9$



Total revenue with ρ_4 and $\gamma = 0.9$



Future trading with $CVaR_{0.05}$ and $\gamma = 0.9$

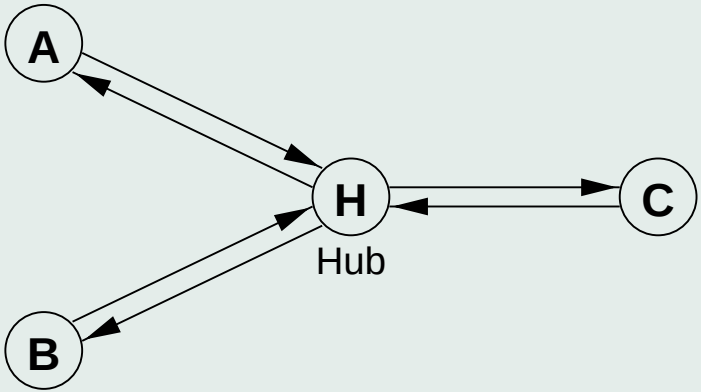


Future trading with ρ_2 and $\gamma = 0.9$

O&D airline revenue management: Numerical results

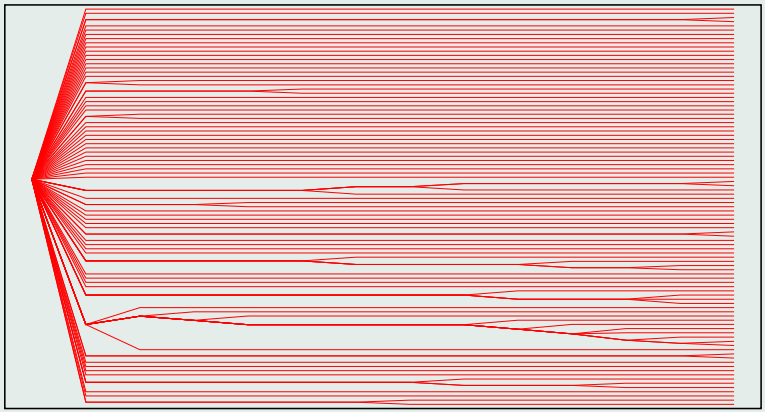
Hub-and-Spokes Network

#Legs	6
#ODIs	12
#Compartments	2
#Fare Classes	6
#POS	1
#DCPs	14



Tree and Size

#Scenarios	95
#Nodes	1022
#cont. Variables	434304
#bin. Variables	73512
#Constraints	442212

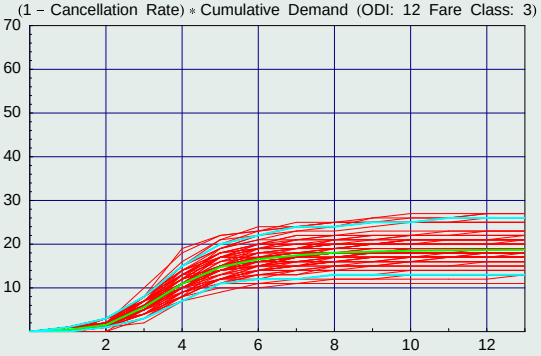
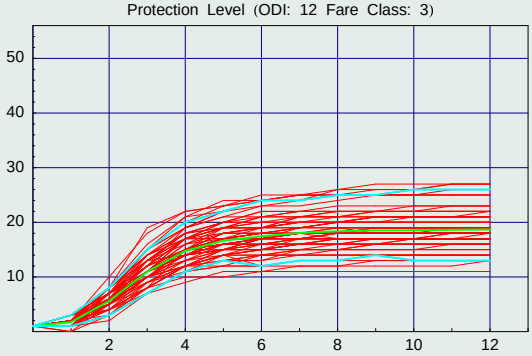
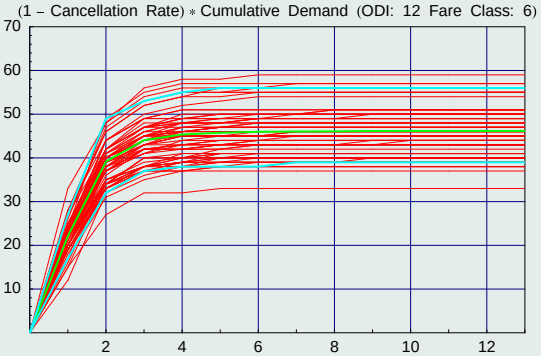
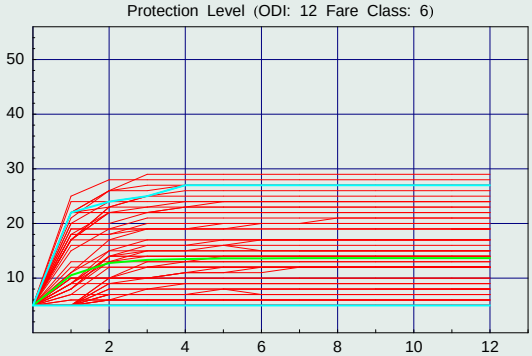


Numerical Results

CPLEX-Results

Version	8.1
MIP Gap	0.001
Solution Status	Optimal
#MIP Nodes passed	0 (root)

Computing times

Total	41.920 s
→ Solving Problem (CPLEX)	36.81 s
→→ CPLEX Presolve Time	13.32 s

(Intel Celeron, 2.0 GHz, Linux)