Portfolio Optimization: A Technical Perspective

Michael Bussieck

MBussieck@gams.com

Franz Nelißen

FNelissen@gams.com

GAMS Software GmbH

http://www.gams.de

GOR Workshop "Financial Optimization and Optimal Pricing Strategies"

BASF, Ludwigshafen, Germany, May 22-23, 2006

Agenda

- GAMS Software and GAMS
- Mean Variance Model
- Adding Business Rules
- Scenario Optimization Models
- Grid Computing

GAMS Dev. / GAMS Software

- *Roots: **Research project** World Bank 1976*
- Pioneer in **Algebraic Modeling Systems** used for economic modeling
- Went **commercial** in 1987
- **Offices** in Washington, D.C and Cologne
- Professional **software tool provider**
- Broad **academic** & **commercial** user base
- Operating in a **segmented niche market**

GAMS

Typical Application* Areas:

- Agricultural Economics
- Chemical Engineering
- Econometrics
- Environmental Economics
- Finance
- International Trade
- Macro Economics
- Management Science / OR
- Micro Economics

- Applied General Equilibrium
- Economic Development
- Energy
- Engineering
- Forestry
- Logistics
- Military
- Mathematics
- Physics

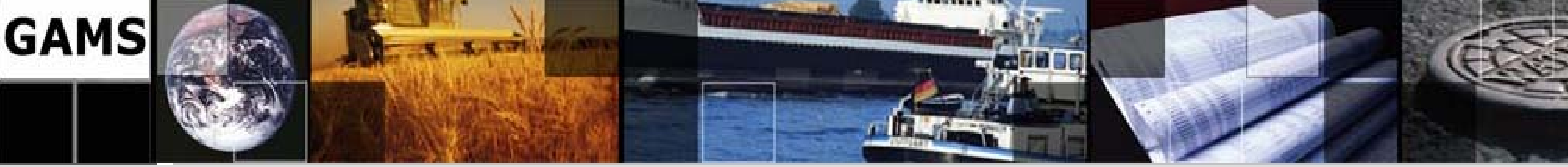
* Illustrative examples in the GAMS Model Library

Algebraic Modeling Language

- Efficient handling of **mathematical optimization problems**
- **Declarative** approach: Algebraic model representation
  - is close to mathematical formulation:
    - Variables, constraints with arbitrary names
    - Sets, indices, algebraic expressions, powerful sparse index and data handling
  - is a self containing and executable description of the mathematical optimization problem
  - contains no hints how to process it
- Also **procedural** elements: Loops, procedures, macros, …

Algebraic Modeling Language cont’d

- Different **Layers** with **separation** of:
  - Model and data: Core model is independent of data and scalable
  - Model, solution methods and solver
  - Model and operating system
  - Model and application
- Wide range of **supported model types**
- Large **libraries** of **example models** and **blue prints** available online



System Overview

Connectivity Tools:

- Uniform Data Exchange: ASCII or **GDX** (ODBC, SQL, XLS, XML)
- Data API
- Ext. Programs
  - EXCEL
  - MATLAB
  - GNUPLOT,..
  - C, Delphi, ..

Interactive | Offline / Batch

User Interfaces

GAMS Compiler and Execution System

Solvers

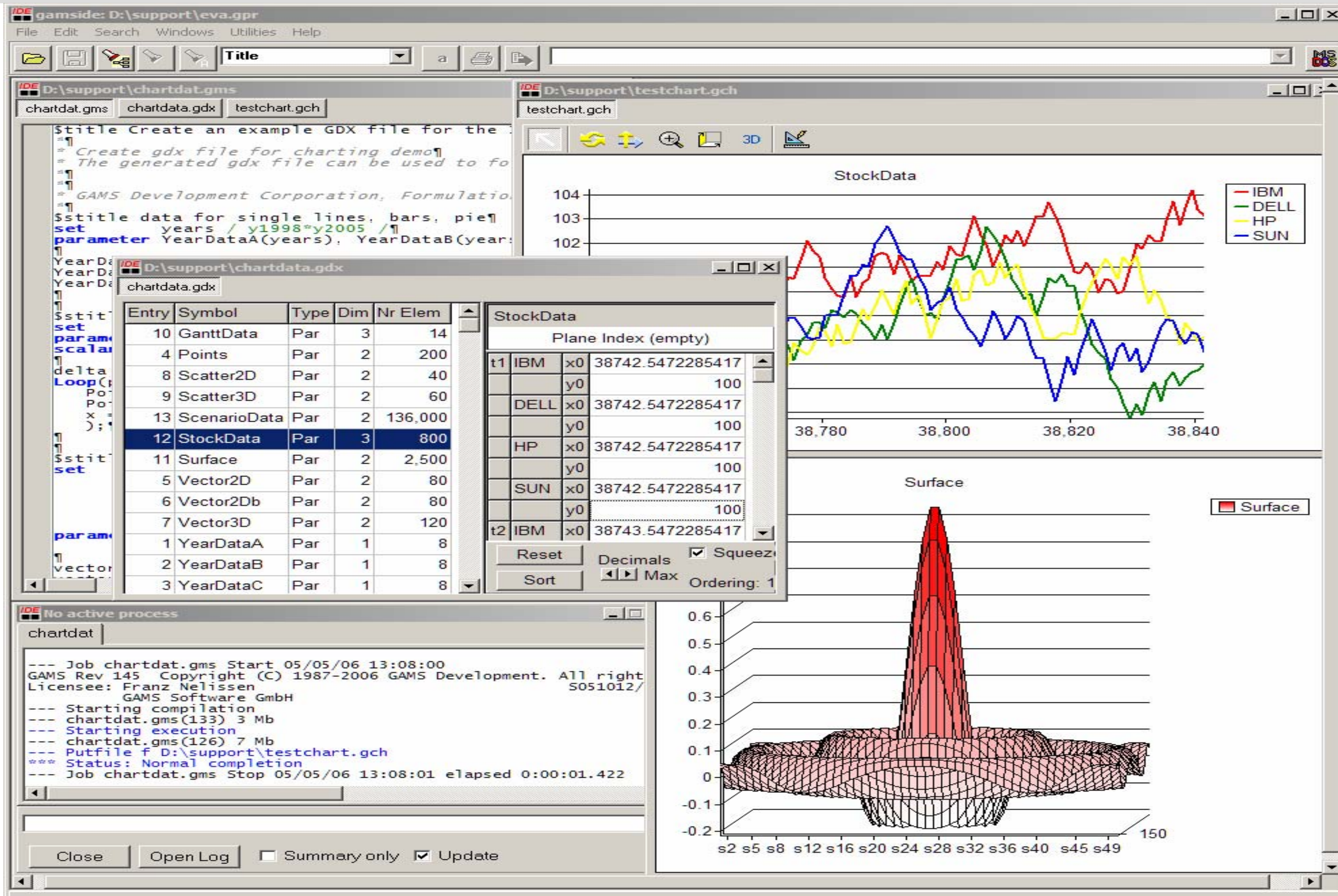
LP-MIP-QCP-MIQCP-NLP-MINLP-CNS-MCP-MPEC
MPSGE, global, and stochastic optimization

Productivity Tools:

- Integrated Development Environment
- Model Debugger and Profiler
- GDX Tools
- Data Browser
- Charting Engine
- Benchmarking
- Deployment System
- Quality Assurance and Testing

BARON, COIN, CONOPT, CPLEX, DECIS, DICOPT, KNITRO, LGO,MINOS, MOSEK, OQNLP, PATH, SNOPT, XA, XPRESS, …

GAMS IDE

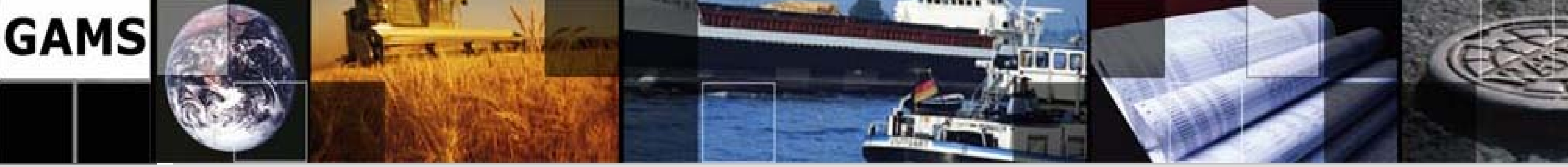

gamside: D:\support\eva.gpr
File Edit Search Windows Utilities Help
Title
D:\support\chartdat.gms
chartdat.gms chartdata.gdx testchart.gch
$title Create an example GDX file for the
* Create gdx file for charting demo
* The generated gdx file can be used to fo
* GAMS Development Corporation, Formulatio
$stitle data for single lines, bars, pie
set years / y1998*y2005 /
parameter YearDataA(years), YearDataB(year
D:\support\testchart.gch
testchart.gch
StockData
IBM
DELL
HP
SUN
104
103
102
38,780
38,800
38,820
38,840
D:\support\chartdata.gdx
chartdata.gdx
Entry Symbol Type Dim Nr Elem
10 GanttData Par 3 14
4 Points Par 2 200
8 Scatter2D Par 2 40
9 Scatter3D Par 2 60
13 ScenarioData Par 2 136,000
12 StockData Par 3 800
11 Surface Par 2 2,500
5 Vector2D Par 2 80
6 Vector2Db Par 2 80
7 Vector3D Par 2 120
1 YearDataA Par 1 8
2 YearDataB Par 1 8
3 YearDataC Par 1 8
StockData
Plane Index (empty)
t1 IBM x0 38742.5472285417
y0 100
DELL x0 38742.5472285417
y0 100
HP x0 38742.5472285417
y0 100
SUN x0 38742.5472285417
y0 100
t2 IBM x0 38743.5472285417
Reset
Sort
Decimals
Max
Ordering: 1
Surface
Surface
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
150
s2 s5 s8 s12 s16 s20 s24 s28 s32 s36 s40 s45 s49
No active process
chartdat
--- Job chartdat.gms Start 05/05/06 13:08:00
GAMS Rev 145 Copyright (C) 1987-2006 GAMS Development. All right
Licensee: Franz Nelissen S051012/
GAMS Software GmbH
--- Starting compilation
--- chartdat.gms(133) 3 Mb
--- Starting execution
--- chartdat.gms(126) 7 Mb
--- Putfile f D:\support\testchart.gch
*** Status: Normal completion
--- Job chartdat.gms Stop 05/05/06 13:08:01 elapsed 0:00:01.422
Close
Open Log
Summary only
Update

Portfolio Optimization Models

- Mean-Variance Model
- *Portfolio Models for Fixed Income*
- Scenario Optimization
- *Stochastic Programming*

The Mean-Variance Model

- **Markowitz** (1952)→Nobel prize 1990
- **Given:** Some investments x_i with historical data:
  - **Expected returns** of investments: μ_i (**Mean** of historical returns)
  - Risk: **Variance** of investments *Qi,j*
- **Goal**: **Balance risk** *r* of portfolio against expected **returns** of portfolio
- **Idea: Minimize variance** *v* of portfolio for a **given target return** *r*



MV Model Algebra

$$Min \sum_{i=1}^{I} \sum_{j=1}^{J} x_i \, Q_{i,j} \, x_j$$

$$s.t. \quad \sum_{i=1}^{I} \mu_i x_i \geq r$$

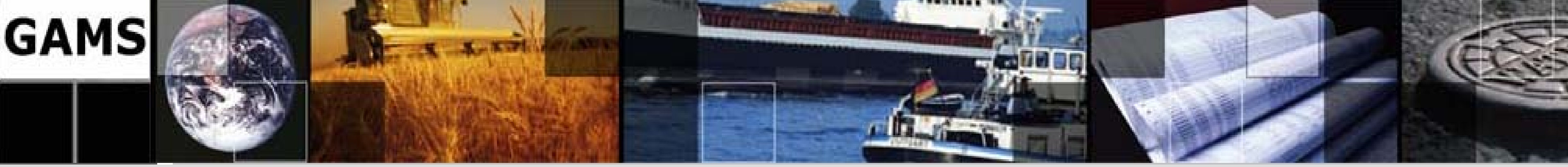
Target return

$$\sum_{i=1}^{I} x_i = 1$$

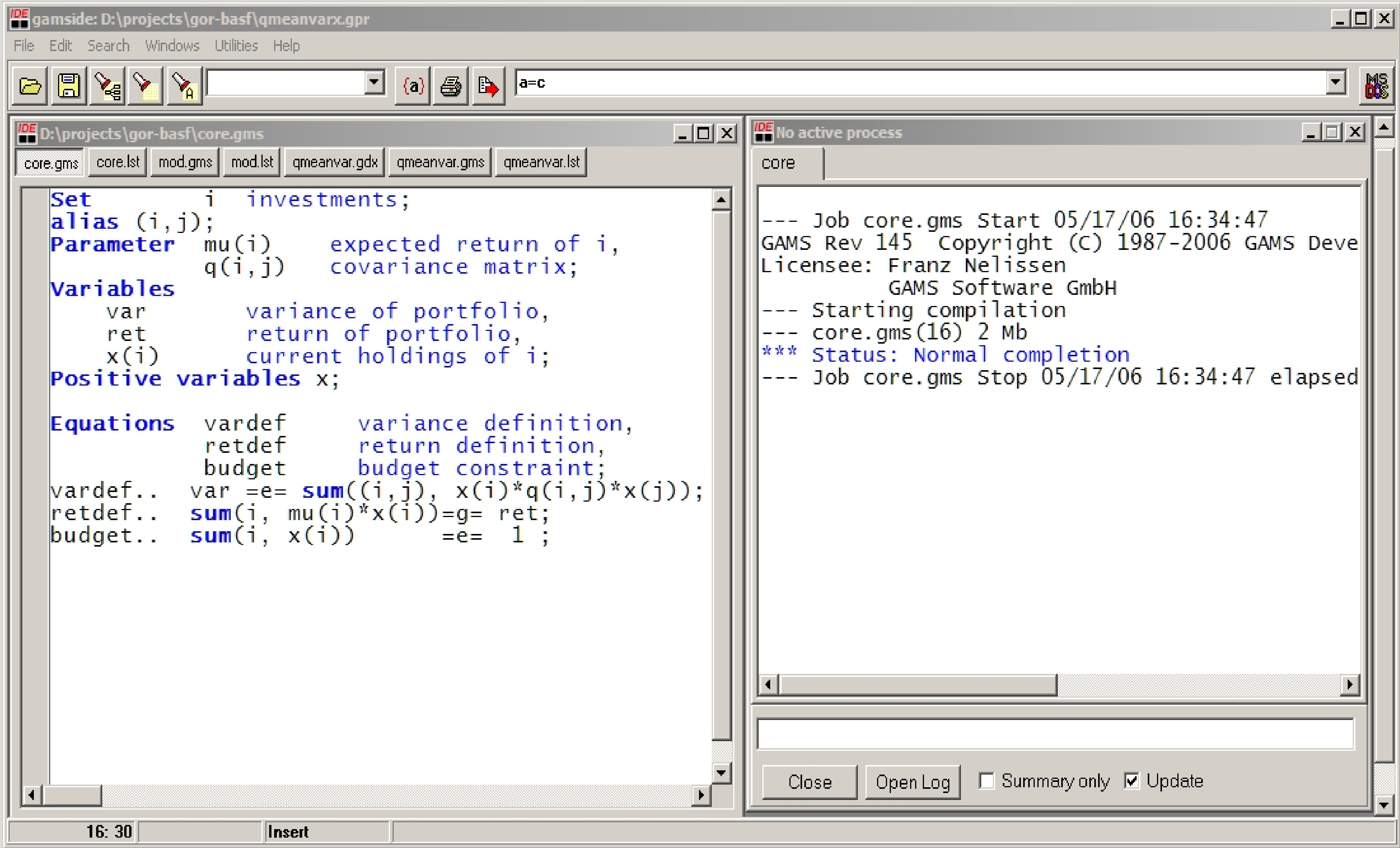
Budget constraint

$$x_i \geq 0$$

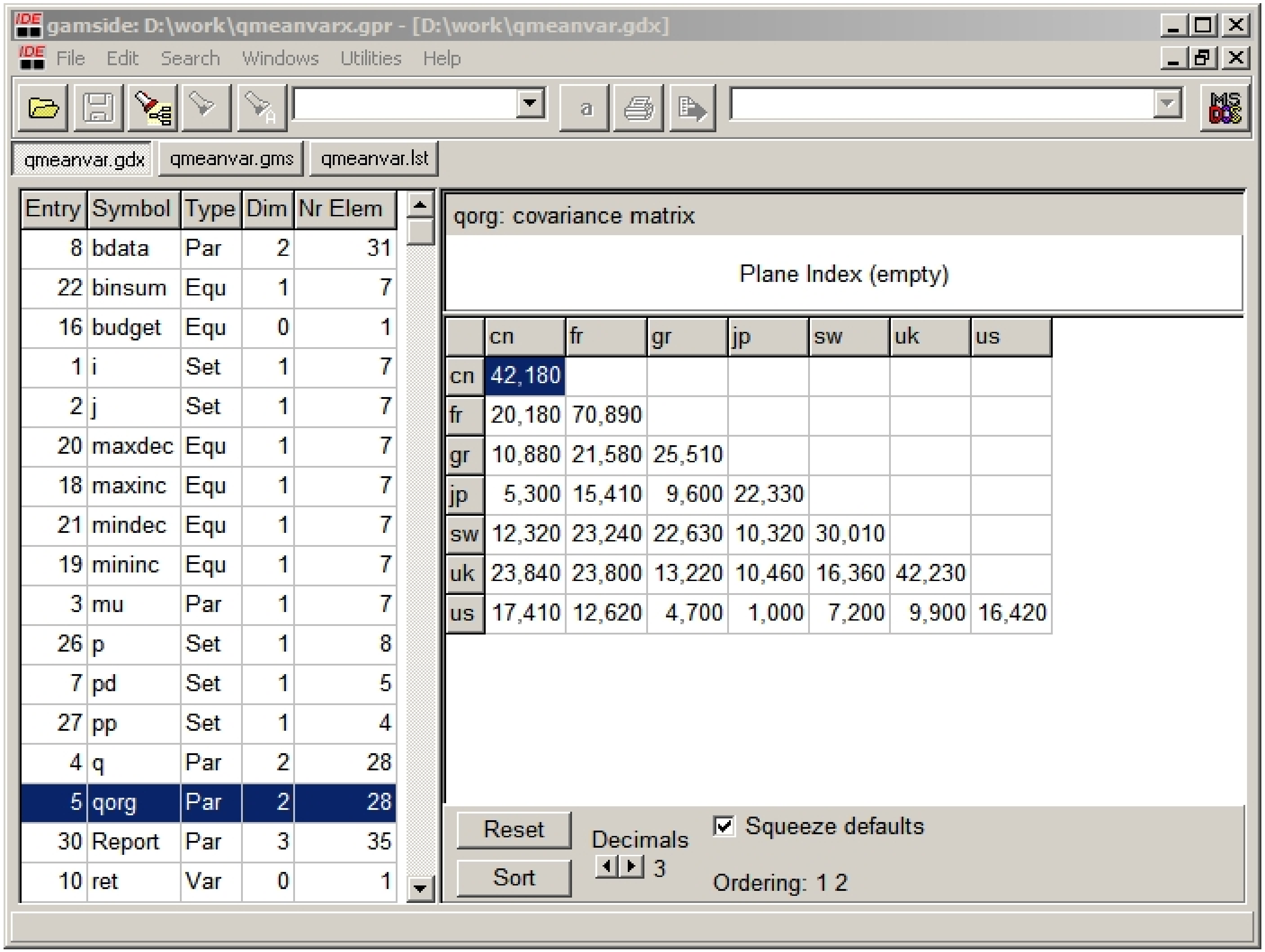
No short sales



Declarative Model



Data: Variance/Covariance Matrix



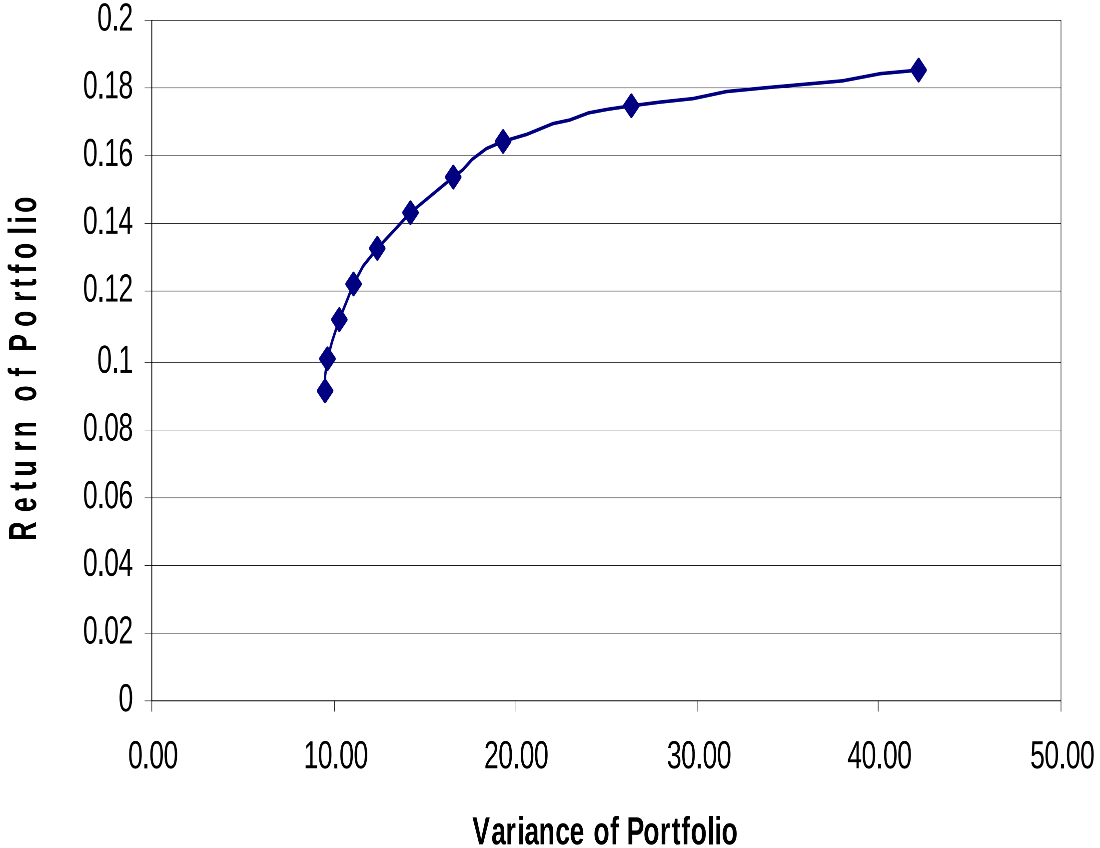
Entry	Symbol	Type	Dim	Nr Elem
8	bdata	Par	2	31
22	binsum	Equ	1	7
16	budget	Equ	0	1
1	i	Set	1	7
2	j	Set	1	7
20	maxdec	Equ	1	7
18	maxinc	Equ	1	7
21	mindec	Equ	1	7
19	mininc	Equ	1	7
3	mu	Par	1	7
26	p	Set	1	8
7	pd	Set	1	5
27	pp	Set	1	4
4	q	Par	2	28
5	qorg	Par	2	28
30	Report	Par	3	35
10	ret	Var	0	1

	cn	fr	gr	jp	sw	uk	us
cn	42,180						
fr	20,180	70,890					
gr	10,880	21,580	25,510				
jp	5,300	15,410	9,600	22,330			
sw	12,320	23,240	22,630	10,320	30,010		
uk	23,840	23,800	13,220	10,460	16,360	42,230	
us	17,410	12,620	4,700	1,000	7,200	9,900	16,420

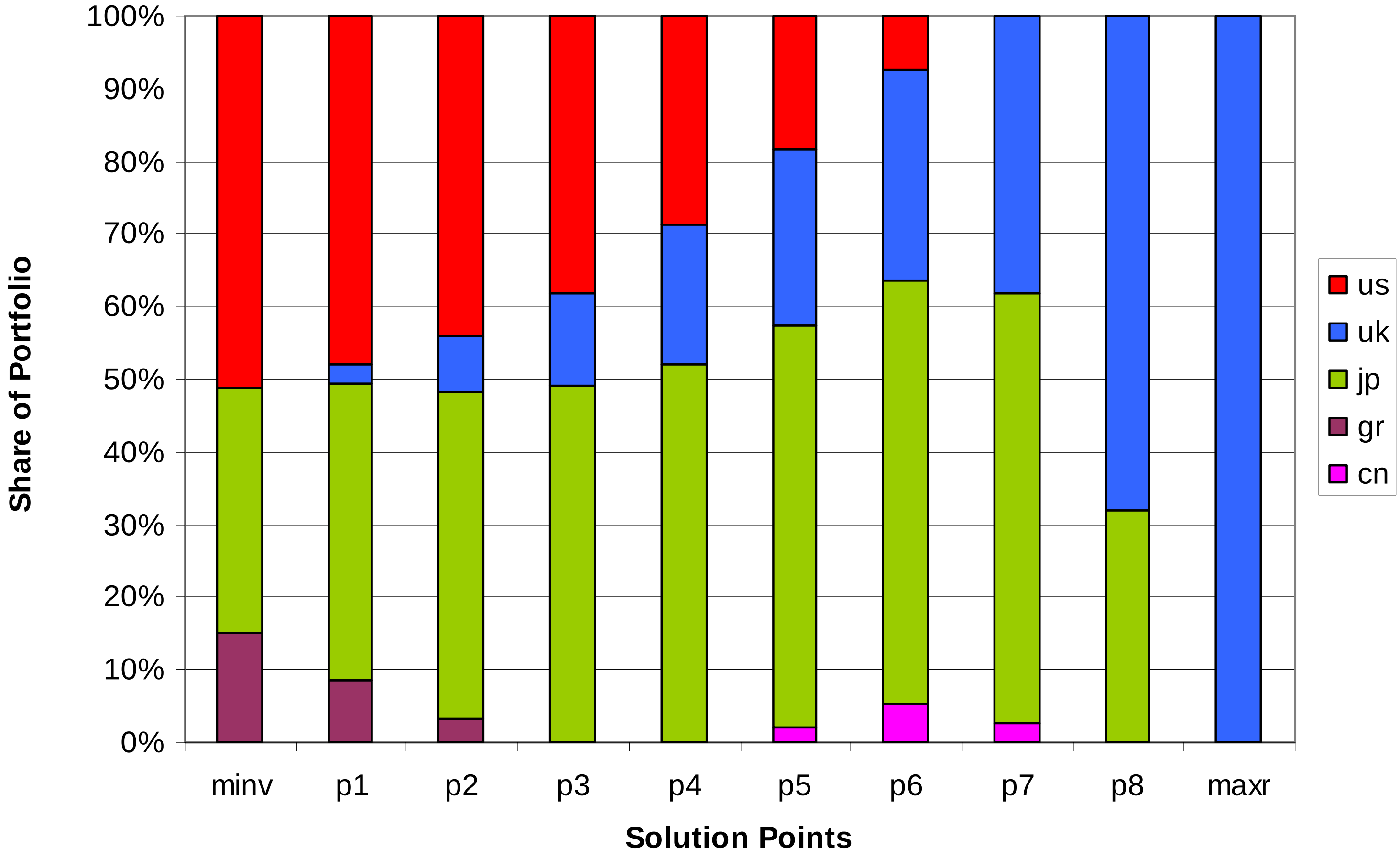
Procedural Elements

```
$gdxin data                                             # get data & setup model
$load i mu q
q(i,j) = 2*q(j,i) ;  q(i,i) = q(i,i)/2;
Model var / all / ;
set p     points for efficient frontier /minv, p1*p8, maxr/,
    pp(p) points used for loop            /      p1*p8      /;
parameter minr, maxr,rep(p,*),  repx(p,i);

solve var minimizing v using qcp;         #find portfolio with minmal variance
minr = r.l; rep('minv','ret') = r.l;
rep('minv','var') = v.l; repx('minv',i) = x.l(i);

solve var maximizing r using qcp;         #find portfolio with maximal return
maxr = r.l; rep('maxr','ret')= r.l;
rep('maxr','var')=v.l;repx('maxr',i)= x.l(i);

loop(pp,                                         #calculate efficient frontier
         r.fx = minr + (maxr-minr)/(card(pp)+1)*ord(pp);
         solve var minimizing v using qcp;
         rep(pp,'ret') =r.l;rep(pp,'var') = v.l;repx(pp,i)= x.l(i);
     );
Execute_Unload 'results.gdx',rep, repx;         # export results to GDX & Excel
Execute 'GDXXRW.EXE results.gdx par=repx rng=Portfolio!a1 Rdim=1';
Execute 'GDXXRW.EXE results.gdx par=rep  rng=Frontier!a1  Rdim=1';
```

Efficient Frontier



Efficient Portfolios



Modeling Issues

- Basic MV-Model: Quadratic model
- GAMS Model type: NLP or QCP
- Solver
  - – NLP Codes (CONOPT, MINOS,...) *or*
  - – QCP Codes (Cplex, Mosek, Xpress)
    - take advantage of special structure
    - include strong machinery from linear programming world (pre-solve techniques)
- Large problem instances can be solved routinely

Incorporating Business Rules

- Institutional or legal requirements
- Additional constraints, which have to be satisfied: Trading restrictions
- Independent of risk model
- Not defined by modeling experts

Simple Trading Restrictions

- Do not change the model type
- Examples:
  - Short selling
  - Risk free borrowing
  - Upper or lower bounds on certain instruments

More Complex Trading Restr.

- Require introduction of integer (binary) variables
- Quadratic model with integer variables
- GAMS model type: MINLP or MIQCP

Cardinality / Threshold Constraint

- Cardinality Constraint: Restricts number of investments y_i in a portfolio:

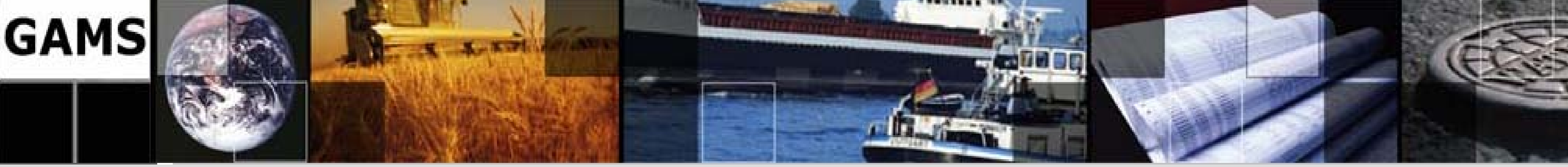
$$\sum_i y_i \leq C, y_i \in \{0,1\}, \; i = 1,\ldots,n$$

- Threshold Constraint: Investments x_i can only be purchased at certain minimum $l_{l,i}$ or maximum $l_{u,i}$:

$$l_{l,i} \leq x_i * y_i \leq l_{u,i}, y_i \in \{0,1\}, i = 1,\ldots,n$$

“Zero or Range”-Constraint

- Revision of existing (not optimized) portfolio
- “Zero or Range”-Constraint: **Either** no trade **or** the trade must stay between pre-defined ranges both for purchase and selling
- Portfolio turnover: The total purchase of investments x_i may not exceed some threshold τ



Trading Restrictions: Data

D:\projects\gor-basf\qmeanvar.gdx

core.gms | core.lst | mod.gms | mod.lst | qmeanvar.gdx | qmeanvar.gms | qmeanvar.lst

Entry	Symbol	Type	Dim	Nr Elem
7	bdata	Par	2	31
21	binsum	Equ	1	7
15	budget	Equ	0	1
1	i	Set	1	7
2	j	Set	1	7
19	maxdec	Equ	1	7
17	maxinc	Equ	1	7
20	mindec	Equ	1	7
18	mininc	Equ	1	7
3	mu	Par	1	7
25	p	Set	1	8

bdata: portfolio data and trading restrictions

Plane Index (empty)

	old	umin	umax	lmin	lmax
cn	0.20	0.03	0.11	0.02	0.20
fr	0.20	0.04	0.10	0.02	0.15
gr		0.04	0.07	0.04	
jp		0.03	0.11	0.04	
sw	0.20	0.03	0.20	0.04	0.10
uk	0.20	0.03	0.10	0.04	0.15
us	0.20	0.03	0.10	0.04	0.20

e.g. cn: either no trade (20%) or new share between 23-31% (u) or between 0-18% (l)

GAMS Formulation

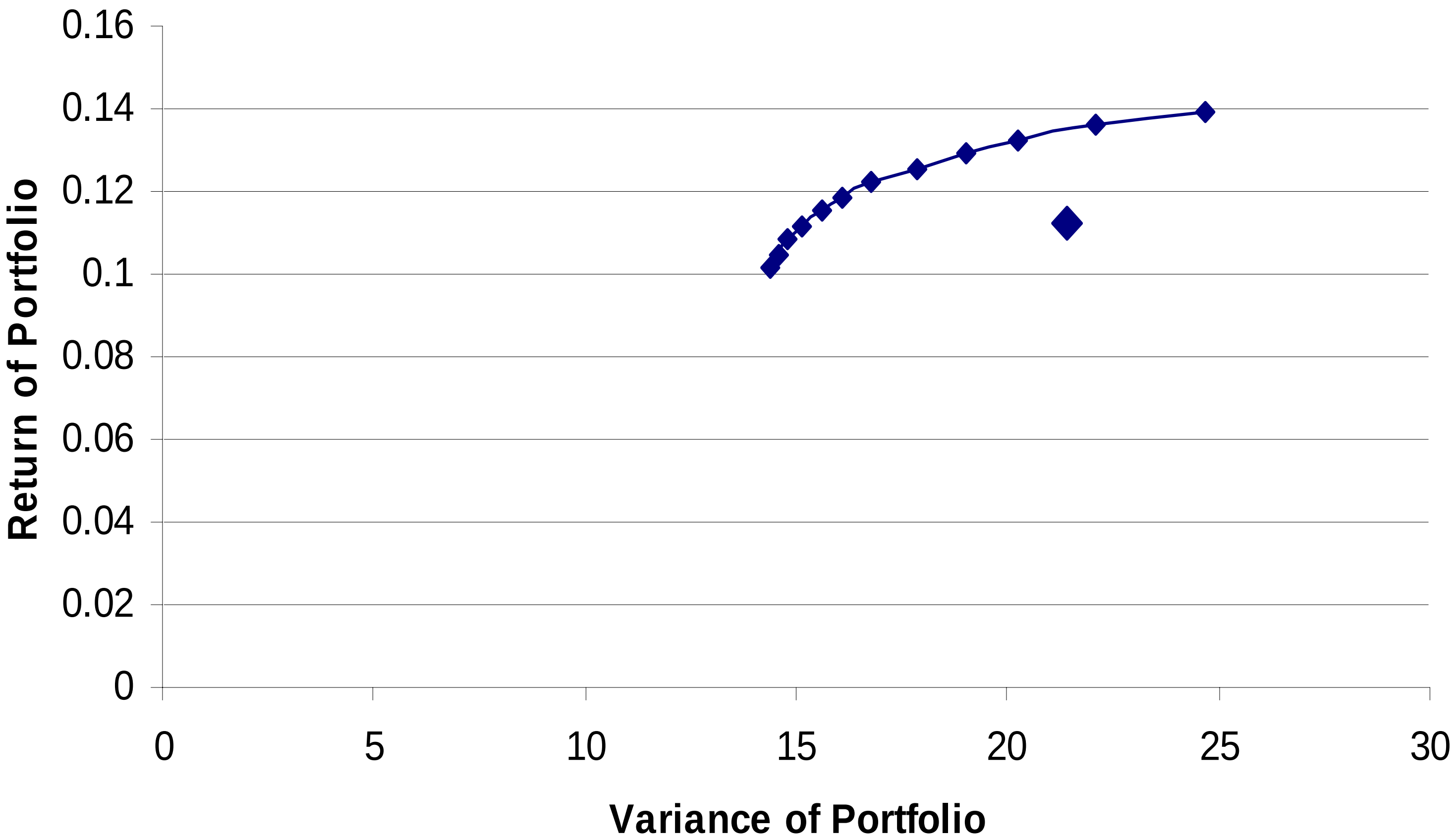
```
Variables
 xi(i)   fraction of portfolio increase,
 xd(i)   fraction of portfolio decrease,
 y(i)    binary switch for increasing current holdings of i,
 z(i)    binary switch for decreasing current holdings of i;
Binary Variables  y, z;
Positive variables xi, xd;
Equations
 xdef(i)    final portfolio definition,
 maxinc(i)  bound of maximum lot increase of fraction of i,
 mininc(i)  bound of minimum lot increase of fraction of i,
 maxdec(i)  bound of maximum lot decrease of fraction of i,
 mindec(i)  bound of minimum lot decrease of fraction of i,
 binsum(i)  restricts use of binary variables,
 turnover   restricts maximum turnover of portfolio;
```

GAMS

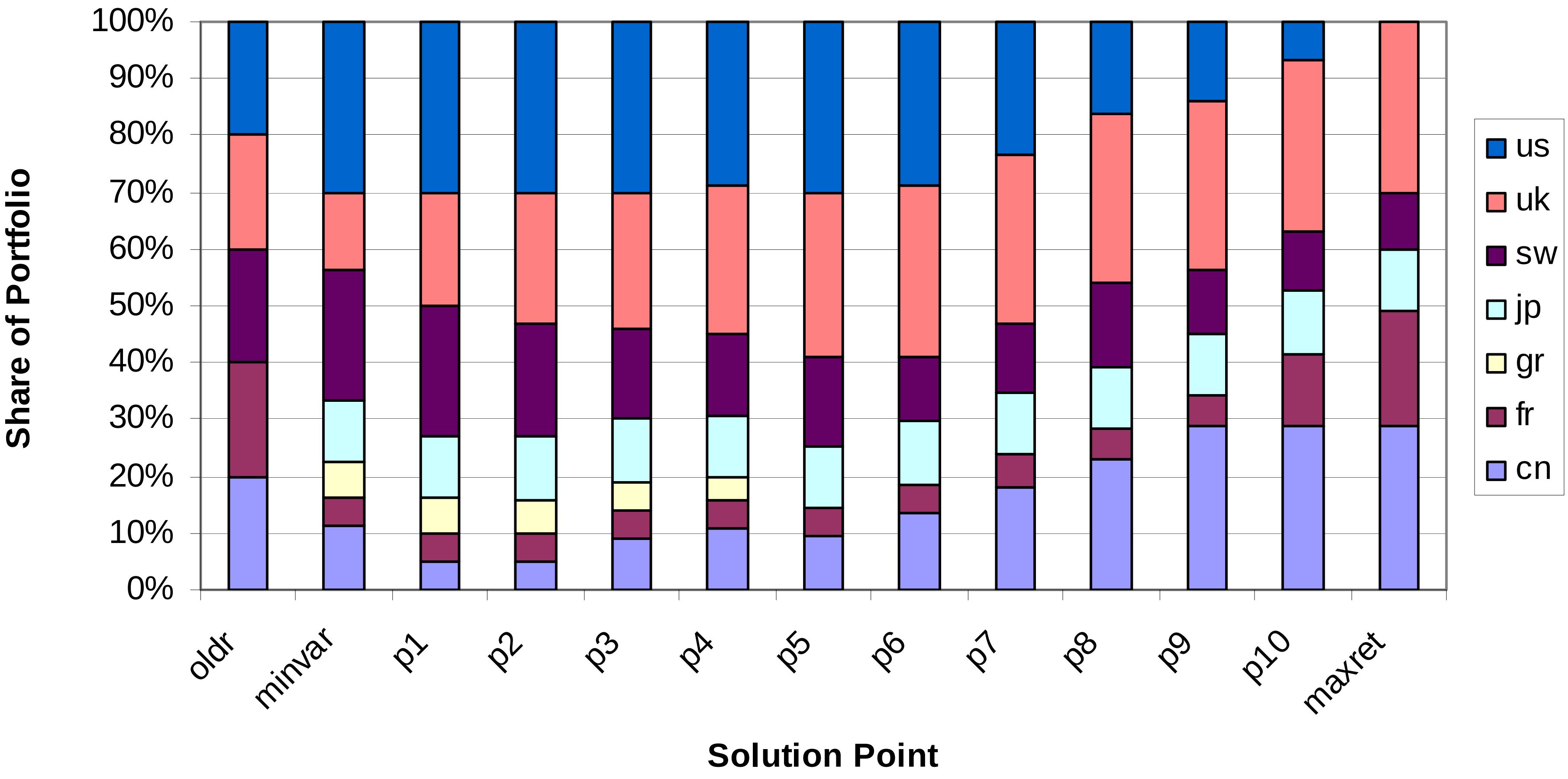
GAMS Formulation cont’d

```
xdef(i)..   x(i)   =e=  bdata(i,'old') - xd(i) + xi(i);

maxinc(i).. xi(i)  =l=  bdata(i,'umax')* y(i);
mininc(i).. xi(i)  =g=  bdata(i,'umin')* y(i);
maxdec(i).. xd(i)  =l=  bdata(i,'lmax')* z(i);
mindec(i).. xd(i)  =g=  bdata(i,'lmin')* z(i);

binsum(i).. y(i) + z(i)        =l= 1;

turnover.. sum(i, xi(i))       =l= tau;
```

Efficient Frontier (τ =0.3)



Portfolios ($\tau = 0.3$)



Limitations of the MV-Approach

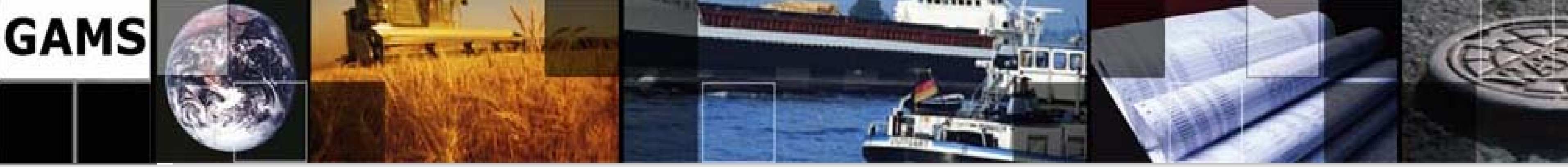
- Quadratic model
- **Risk Measure**: Variance not appropriate for asymmetric and skewed distributions
- **Data**: Estimation errors in the covariance matrix
- **Robustness**: MV efficient portfolios are not robust to small data changes
- Single period model

Scenario Optimization

- Captures complex interactions between multiple risk factors using scenarios
- Scenarios can be quite general describing different kinds of risk
- Scenario generation methods problem specific
- Models are solved over all scenarios
- Different methods of risk measurement

Scenario Optimization Models

- Mean Absolute Deviation Models
- Index Tracking Models
- Expected Utility Models
- VAR Models (linear Version: CVAR)



Mean Absolute Deviation - Model

$$Minimize \sum_{l \in \Omega} p^l \left| V(x; P^l) - V(x; \bar{P}) \right|$$

$$subject\, to: \sum_{i=1}^{n} \bar{P}_i \, x_i \geq \mu V_0$$

Target Value

$$\sum_{i=1}^{n} P_{0,i} x_i = V_0$$

Budget constraint

$$x_i \geq 0$$

Linear Version

$$Minimize \sum_{l \in \Omega} p^l y^l$$

$$subject\ to: y^l \geq V(x; P^l) - V(x; \bar{P}) \quad \forall\, l \in \Omega$$

pos. Dev

$$y^l \geq V(x; \bar{P}) - V(x; P^l) \quad \forall\, l \in \Omega$$

neg. Dev

$$\sum_{i=1}^{n} \bar{P}_i\, x_i \geq \mu V_0$$

Target Value

$$\sum_{i=1}^{n} P_{0,i} x_i = V_0$$

Budget constraint

$$x_i \geq 0; y^l \geq 0$$

GAMS Formulation

```
VARIABLES x(i)       "Current Holdings of Stock I in monetary Units",
          Value(l)  "Final Portfolio Value in Scenario l",
          ExpValue  "Expected final Portfolio Value",
          MAD       "Mean Absolute Deviation",
          Y(l)      "Measures deviation in scenario l";
POSITIVE  VARIABLES x(i), y(l);

EQUATIONS MADDef    "Mean Absolute Deviation of Portfolio"
          posDef(l) "Positive Deviation in Scenario l",
          negDev(l) "Negative Deviation in Scenario l",
          ValDef(l), ExpValDef,
          ExpValLimit, BudgetDef;

MADDef      ..  MAD           =E= sum(l, prob(l) * y(l));
posDev(l)   ..  y(l)          =G= Value(l) - ExpValue;
negDev(l)   ..  y(l)          =G= ExpValue - Value(l);
ValDef(l)   ..  Value(l)      =E= sum(i, (1+ScenRet(i,l)) * x(i));
ExpValDef   ..  ExpValue      =E= sum(l, prob(l) * Value(l));
ExpValLimit ..  ExpValue      =G= Mu * Budget;
BudgetDef   ..  sum(i, x(i)) =E= Budget;
```

Modeling Issues

- Linear Model
- Same results as MV-Model if returns are multivariate normally distributed
- MIP Model, if (complex) business rules
- Variations:
  - Weights on deviations
  - Left (right) semi-absolute deviation

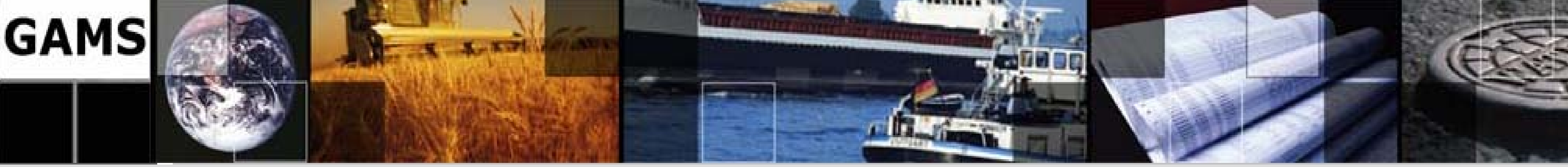
More Theory and Templates

- **Practical Financial Optimization** (forthcoming) by S. Zenios
- **A Library of Financial Optimization Models** (forthcoming) by A. Consiglio, S. Nielsen, H. Vladimirou and S. Zenios
- **Financial Optimization** by S. Zenios (ed.)
- **Online:**
  - **Course Notes „Financial Optimization“:** http://www.gams.com/docs/contributed/financial/
  - **GAMS Model Library:** http://www.gams.com/modlib/libhtml/subindx.htm

Summary

- Portfolio Optimization is one of the success stories in OR
- Rich set of different risk models available
- Large problem instances can be modeled and solved with standard software tools
- Integration of business rules increases model complexity, but is essential for acceptance of advanced techniques
- Algebraic Modeling Languages are powerful and reliable tools for the rapid development and implementation of these models

“*If the only tool you have is a hammer, you will see every problem as a nail.*” (Abraham Maslow)



New Opportunities

- Considerer high throughput computing
- How to convert from serial to parallel and distributed computing
- High Throughput Computing via the Condor system and the SUN Grid Engine connected to GAMS
- Multi CPU desktop systems available
- GAMS introduced an experimental grid computing facility

What is Grid Computing?

- A pool of connected computers managed and available as a common computing resource
  - Allows parallel task execution
  - Allows effective sharing of CPU power
  - Licensing issues
  - Scheduler handles management tasks
  - Can be rented or owned in common
  - E.g. Condor, Sun Grid Engine, Globus

GAMS

Economics of Grid Computing

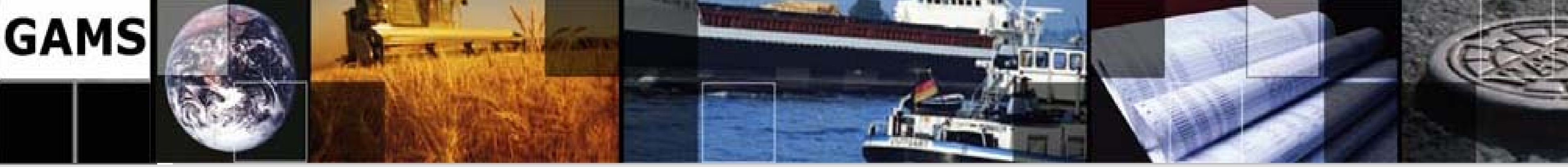
- Yearly cost, 2-CPU workstation: $5200
  - Hardware - $1200
  - Software - $4000
- Hourly cost on the grid: $2
  - $1/hour for CPU time (to grid operator)
  - $1/hour for software (GAMS, model owner)
- 1 workstation == 50 hrs/week grid time
- Up-front vs. deferred, as-needed costs

Use a GAMS Grid

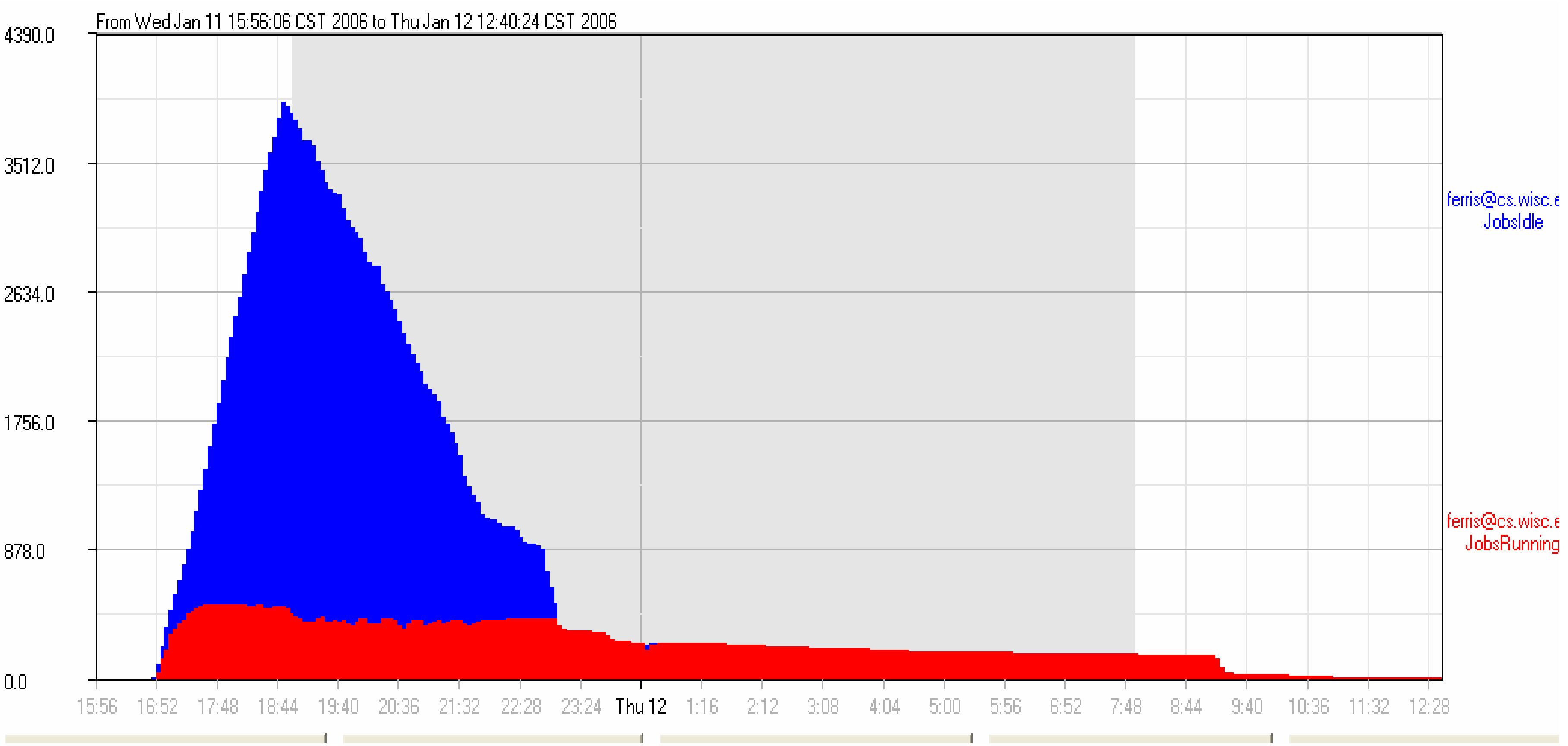
- Solve the scenarios in parallel, e.g.
  - Sequential time: 50 hours
  - 200 CPUs: 15 minutes
- Cost is $100
- No programming required (almost)
- Model stays maintainable
- Separation of model and solution maintained

Results for 4096 MIPS

- Submission start Jan 11 at 16:00 pm
- All job submitted by Jan 11 at 23:00 pm
- All jobs returned by Jan 12, 12:40 pm
  - 20 hours wall time, 5000 CPU hours
  - Peak number of CPUs: 500
- Different Instance:
  - 24 hours wall time, 3000 CPU hours



Condor Pool Statistics



Serial Solve Loop

```
Loop(p(pp),
   ret.fx = rmin + (rmax-rmin)/(card(pp)+1)*ord(pp) ;
   Solve minvar min var using miqcp ;
   xres(i,p)          = x.l(i);
   report(p,i,'inc') = xi.l(i);
   report(p,i,'dec') = xd.l(i) );
```

Solve Submit Loop

Parameter **h(p)** store the instance handle;

minvar.**solvelink = 3**; ! turn on grid option

Loop(p(pp),

ret.fx = rmin + (rmax-rmin)/(card(pp)+1)*ord(pp) ;

Solve minvar min var using miqcp ;

h(pp) = minvar.handle); ! save instance handle

Solution Collection Loop

Repeat

loop(p(pp)$h(p),

if(**handlestatus(h(p))=2**,

minvar.handle = h(p); **execute_loadhandle** minvar;

xres(i,p)=x.l(i); report(p,i,'inc')=xi.l(i); report(p,i,'dec')= xd.l(i)

display$**handledelete(h(p))** 'Could not remove handle';

h(p) = 0)) ; ! indicate solution is loaded

if(card(h), execute 'sleep 1');

until card(h) = 0 or timeelapsed > 100;

GAMS

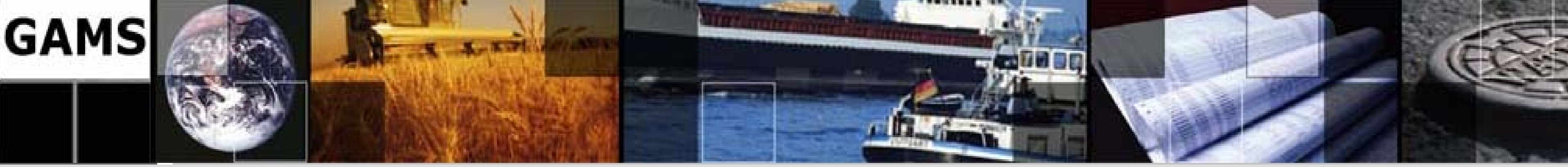
Conclusions

- Massive parallel and distributed computing environments are becoming available (SUN just introduced a 5000 node network in the US giving 100 hours away for free for experiments).
- Simple language extensions in existing modeling systems provide easy access.
- Today's modeling languages are well suited to experiment with coarse grain parallel approaches for solving difficult problems.
  - Latest Example: Ferris & Bussieck: Solving three previously unsolved problems (timtab-2, roll3000, and (*swath*)) from MIPLIB

The End

Thank you!

… Questions?



Contacting GAMS

- Europe:

  GAMS Software GmbH

  Eupener Str. 135-137

  50933 Cologne

  Germany

  Phone: +49 221 949 9170

  Fax: +49 221 949 9171

  Http://www.gams.de

- USA:

  GAMS Development Corp.

  1217 Potomac Street, NW

  Washington, DC 20007

  USA

  Phone: +1 202 342 0180

  Fax: +1 202 342 0181

  Http://www.gams.com