

# Ziena Optimization, Inc.

*Experts in nonlinear optimization*

[www.ziena.com](http://www.ziena.com)



# Ziena Background

- ◆ Incorporated: June 2001
- ◆ Founders: 3 professors + 1 recent Ph.D.
- ◆ Product: **KNITRO**
  - Software to solve nonlinear optimization problems
  - Developed by professors/students at Northwestern University
  - Owned by Northwestern University
  - Exclusive and perpetual license agreement between NU and Ziena signed in February 2002

# What is optimization?

- ◆ Apply mathematical/analytic techniques to uncover the “best” solution to difficult problems involving many decision variables and constraints
- ◆ Two-stage process:
  - Model the problem mathematically  
(AMPL, GAMS, . . .)
  - Solve the mathematical problem  
(KNITRO, MINOS, SNOPT, CONOPT, . . .)

# Linear vs. Nonlinear

## ◆ Linear

- Planning & scheduling problems
- Routing & transportation problems
- Easier – more established
  - ◆ ILOG (CPLEX), Dash Optimization (Xpress)

## ◆ Nonlinear

- Many engineering problems
- Much more difficult – less established
  - ◆ Ziena (KNITRO), etc.

# Optimization software market

- ◆ Annual revenue from sales/services approaching **\$200 million** and growing
- ◆ Bulk of the market for **linear**
- ◆ We specialize in **nonlinear**
  - Much more difficult & complex
  - No dominant developer or vendor
  - Market smaller, less defined

# Nonlinear applications

- ◆ Network design and operations
- ◆ Dynamic pricing
- ◆ Revenue management
- ◆ Mechanical engineering
- ◆ Chemical engineering
- ◆ Circuit design
- ◆ Finance/portfolio management
- ◆ Shape optimization
- ◆ Trajectory optimization

# European customers (corporate)

- ◆ Air Liquide
- ◆ Bouygues Telecom
- ◆ Busarello+Cott+Partner, Inc.  
(Swiss power systems engineering company)
- ◆ Gas du Sud (French gas company)
- ◆ Michelin
- ◆ SIAAP (French water utility)
- ◆ Tractebel S.A (Belgian energy company)
- ◆ *Distributed in Europe by Artelys, S.A. (Paris)*

# European customers (academic)

- ◆ CERT-ONERA, Control System Department (France)
- ◆ École des Mines de Paris
- ◆ ENSEEIHT (Toulouse, France)
- ◆ Fraunhofer Institute (Germany)
- ◆ Université Paul Sabatier, Faculté de Médecine Purpan (France)
- ◆ University of Coimbra, Department of Mathematics (Portugal)



# Other large customers include . . .

- ◆ ExxonMobil
- ◆ Citigroup

# Business plan — funding

- ◆ Phase I (2001-2003)
  - Founders put in some capital
  - Preliminary software sales
  - Phase I SBIR grant obtained Jan 2003
- ◆ Phase II (2004-2006)
  - Continued sales/business partnerships
  - Phase II SBIR grant obtained June 2004
- ◆ Phase III (2006-)
  - Self-sufficient from revenues

# Business plan — revenues

## ◆ KNITRO Software

- Direct Sales
- Maintenance contracts
- Distributors
- Mathematical software vendors
- Value Added Resellers

## ◆ Consulting

## ◆ Sales of modeling software

- AMPL

# Business partnerships

- ◆ Artelys S.A. (European distributor)
- ◆ Frontline Systems (Excel “solver”)
- ◆ Tomlab (MATLAB interface)
- ◆ AMPL Optimization, LLC
- ◆ GAMS Development Corporation
- ◆ **VAR Agreements:**
  - Tractebel S.A.
  - Busarello+Cott+Partner, Inc.

# Long-term goals

- ❖ Run a successful/profitable small-medium size business **or ...**
- ❖ Expand and grow and compete on a large scale **or ...**
- ❖ Sell to one of the big **linear** optimization companies



# **KNITRO**

A software package for optimization

# KNITRO

- ◆ A package for continuous optimization
- ◆ State-of-the-art optimization techniques developed over the last 10 years at Northwestern University
  - ETR 1992
  - NITRO mid 1990s
  - KNITRO 1.0 2000-2001
  - KNITRO 2.0 January 2002
  - KNITRO 3.0 April 2003
  - KNITRO 4.0 October 2004

# KNITRO Strengths

- ◆ **Robust**
- ◆ Efficient
- ◆ Flexible
- ◆ Large-scale
- ◆ Especially good for nonlinear constraints
- ◆ Supported by theory
- ◆ **Commercially supported**



# Problem types

- ◆ Unconstrained
- ◆ Bound constrained
- ◆ Equality constrained
- ◆ Nonlinear equations
- ◆ Least squares
- ◆ Linear and quadratic programs
- ◆ General nonlinear problems (large)

# Problem form

- ◆ KNITRO solves problems that can be written as

$$\begin{array}{ll} \text{Minimize} & f(x) \\ \text{Subject to} & h(x) = 0 \\ & g(x) \geq 0 \end{array}$$

- ◆  $x$  must be continuous
- ◆  $f$ ,  $h$  and  $g$  must be smooth
- ◆ No convexity requirement

# Algorithms

- ◆ Interior-point / barrier
  - **KNITRO/InteriorCG**  
(handles large/dense Hessians)
  - **KNITRO/InteriorDirect**  
(handles ill-conditioned problems)
- ◆ Active-set SLQP (new October 2004!)
  - **KNITRO/Active** (good for warm starts)
- ◆ Trust-region approach
- ◆ Supported by global convergence theory

# Interior algorithms: Example 1

## ◆ CVXQP2

- $n=10,000$ ,  $m=2,500$  + bounds
- $\text{nnzH} = 40,000$
- 99.6% of time spent factoring in **Direct**

| Code                  | iters | time | time/iter |
|-----------------------|-------|------|-----------|
| <b>InteriorCG</b>     | 11    | 401  | 36.5      |
| <b>InteriorDirect</b> | 14    | 2638 | 188.4     |

# Interior algorithms: Example 2

## ◆ BQPGAUSS

- $n = 2003$ , bound-constrained
- Hessian not expensive but ill-conditioned

| Code           | iters | time | time/iter |
|----------------|-------|------|-----------|
| InteriorCG     | 27    | 1310 | 48.5      |
| InteriorDirect | 19    | 3    | 0.16      |

# New algorithm

## ◆ Active-set SLQP

- Alternates linear programs and equality-constrained quadratic programs
- Built on top of simplex solver

## ◆ Advantages

- Crossover techniques
- Better active-set information
- Warm starts

# KNITRO interfaces

## ◆ C/C++/Fortran

- Easily integrated within existing applications via callable library

## ◆ AMPL

- Flexible and powerful syntax
- Derivatives computed automatically
- Focus on modeling and analysis of results
- Ideal for prototyping

## ◆ MATLAB (through Tomlab)

## ◆ GAMS (available soon)

# First derivative options

- ◆ User or modeling language provides exact derivatives
- ◆ KNITRO computes finite difference derivatives (forward or centered)
- ◆ Derivatives can be checked using finite differencing



# Second derivative options

- ◆ User or modeling language provides exact derivatives
- ◆ User or modeling language provides exact Hessian-vector products (KNITRO/InteriorCG, KNITRO/Active)
- ◆ KNITRO computes Hessian-vector products via finite differencing (KNITRO/InteriorCG, KNITRO/Active)
- ◆ Dense quasi-Newton (BFGS or SR1)
- ◆ Limited-memory BFGS

# Feasible Option

- ◆ By default constraints may be violated during the optimization process
- ◆ **Feasible option** enforces feasibility with respect to inequalities given initial point satisfying inequalities
- ◆ Constraints may be undefined outside feasible region
- ◆ Allows early termination with feasible solution

# Solver Options comparison

|                          | KNITRO | MINOS   | SNOPT   | LOQO | IPOPT | GRG |
|--------------------------|--------|---------|---------|------|-------|-----|
| Large-scale<br>(100,000) | x      | X smrsp | X smrsp | x    | x     |     |
| iterative                | x      |         | ?       |      |       |     |
| direct                   | x      | x       | x       | x    | x     | x   |
| Exact Hessian            | x      |         |         | x    | x     | ?   |
| Quasi-Newton             | x      | x       | x       |      | ?     | x   |
| Hessian-vector           | x      |         |         |      |       |     |
| Feasible                 | x      |         |         |      |       | ?   |
| Interior                 | x      |         |         | x    | x     |     |
| Active-set               | x      | x       | x       |      |       | x   |
| Commercially supported   | x      |         |         |      |       | ?   |

# Future Developments

- ◆ Add marketing / project management staff position
- ◆ Mixed-integer nonlinear optimization

# KNITRO future developments

## KNITRO Optimization Package

