

Topology Control and Routing in Mobile Ad Hoc Networks

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Introduction -- Mobile Ad Hoc Networks (MANET)

- **Formed by mobile wireless hosts**
- **Each node may act as a router**
- **Routes between nodes may contain multiple hops**
- **Pre-existing infrastructure is not required**



Introduction – Challenges

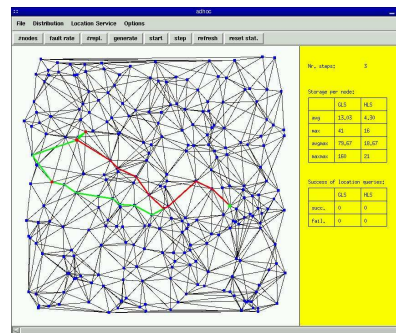
- **Self-construction, self-maintenance**
- **Scalability**
- **Reliability**
- **Resource efficiency**
 - **Energy**
 - **Storage**
- **Fault- / disaster-tolerance**
 - **Tolerates simultaneous (node/link) faults**
 - **Fast reconfiguration on detection of faults**



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Applications

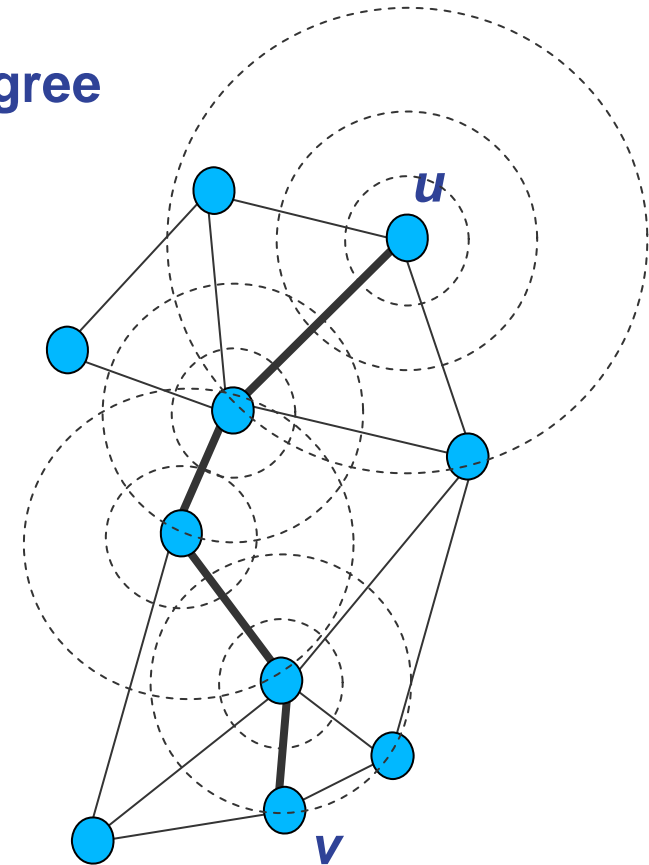
- Extension for existing cellular networks
- Rooftop networks, hybrid networks
- Network level support for position based services
- Sensor networks
- Inter-vehicle networks



Demonstrator

Focus I – Topology Control

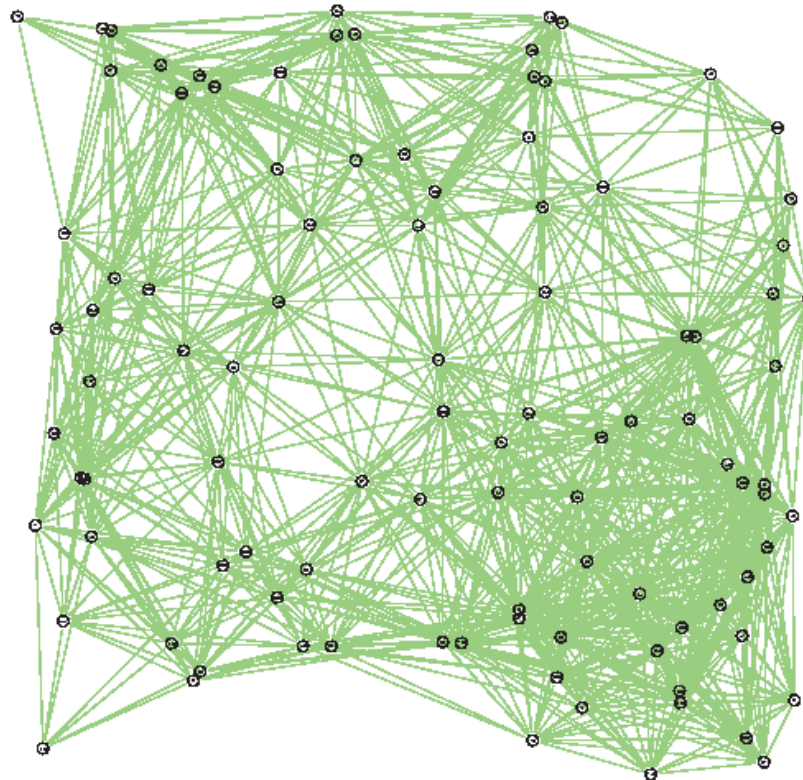
- **Sparse topologies, low node degree**
Storage efficiency
- **Short and low energy paths**
Energy: battery lifetime
health aspects
- **Low congestion path system**
Reliability
- **Efficient distributed construction and maintenance**
Scalability
Fault tolerance



Focus I – Topology Control

Example of No Topology Control

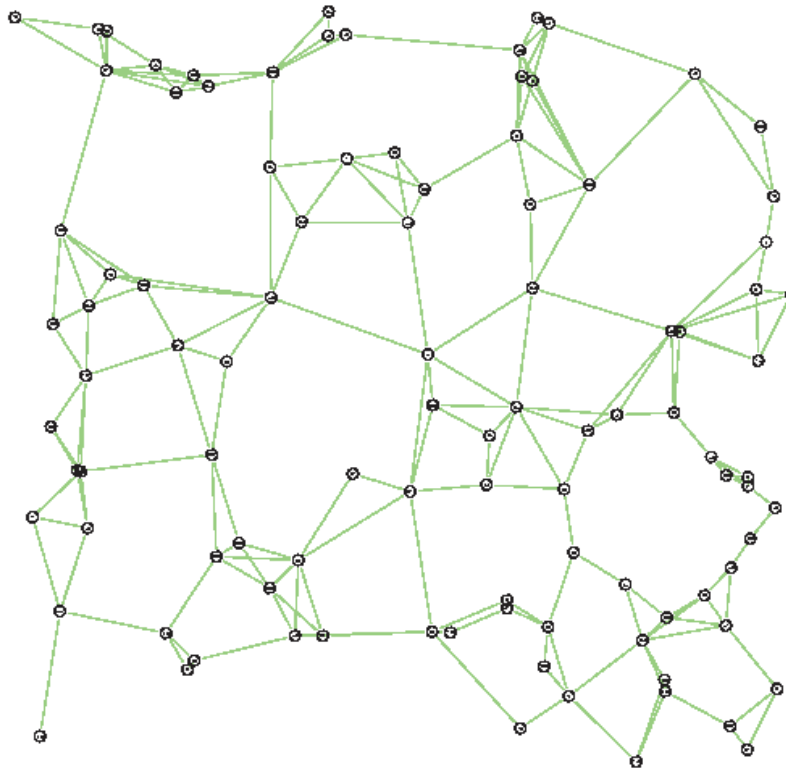
with maximum transmission radius R (maximum connected node set)



- High energy consumption
- High interference
- Low throughput

Focus I – Topology Control

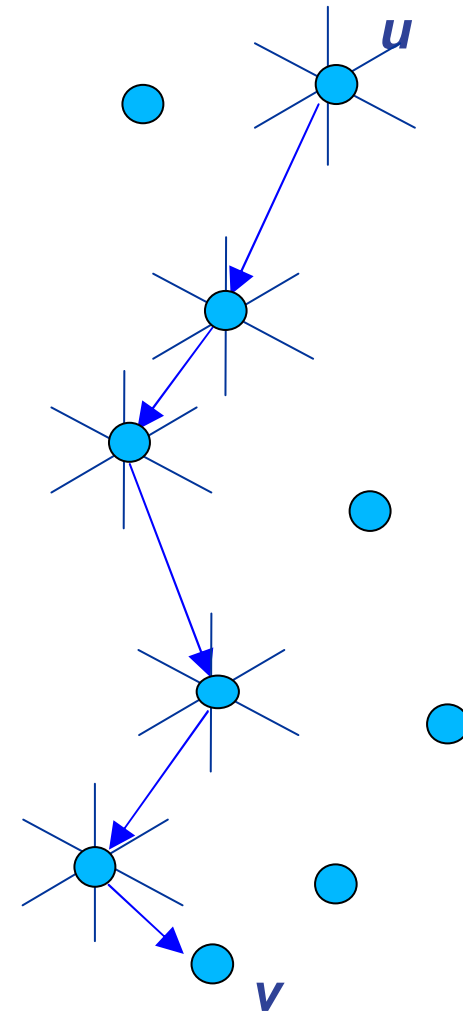
Example of Topology Control



- **Global connectivity**
- **Low energy consumption**
- **Low interference**
- **High throughput**

Focus II - Position Based Routing

- Packets are forwarded “on-the-fly” based on the geographic position of the
 - current node,
 - neighbors of the current node,
 - destination node
- Routing table is not required
 - Storage efficiency, low update costs
- Suited for networks with
 - high node velocity
 - frequent topology changes
- Immediate support of routing to a
 - geographic area
 - (near to) a position
- How can the source discover the position of the destination?



Focus III – Distributed Location Services

Centralized solution - Problems

- Each node has to know the position of the node, which provides the location service (chicken-egg-problem)
- Very high traffic at the location servers

Distributed location service - Desirable properties

- Load is distributed evenly over all nodes
- Low storage and communication costs at the nodes
- Short location discovery paths
- Fault- / disaster-tolerance



Own Contributions*

Topology Control

- **Fundamental results (upper&lower bounds, tradeoffs)**
 - **degree, energy, congestion, maintenance overhead**
 - **dealing with mobility**

Location Services

- **HLS – new disaster tolerant, storage efficient distributed location service**



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Model

Communication Model

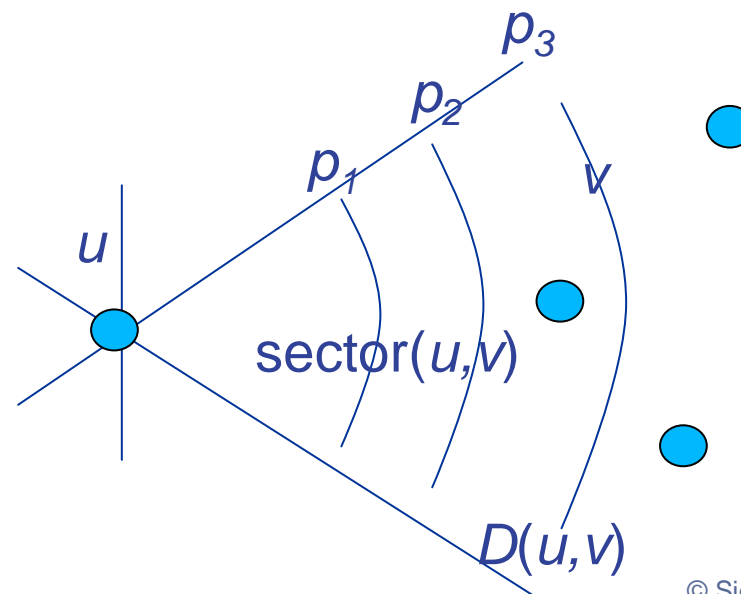
- **Nodes enter or leave**
- **Types of incoming signals:**
 - no signal,
 - interference,
 - clear signal (one frequency)
- **Two types of interferences**
 - uni-directional and bi-directional



Model

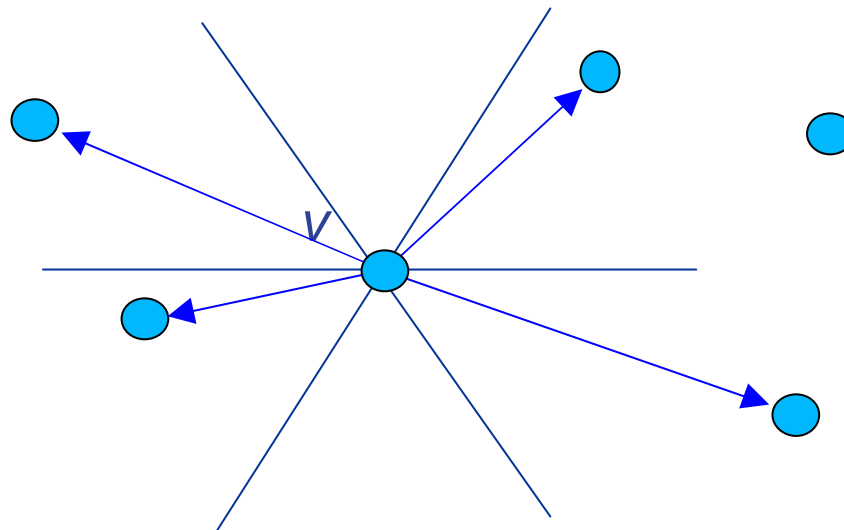
Hardware Model

- Adjustable transmitting power, discrete choices: p_1, \dots, p_s
- k sending and receiving devices (antenna) per node
 - are able to work independently in parallel
 - define sectors
- Orientation and location of the nodes is unknown



Network Topologies

Yao graph [Yao 82]: Around each node $v \in V$, the plane is divided in sectors of an angle $\theta \leq \pi/3$



Each node is connected to its nearest neighbor in each sector by a link:

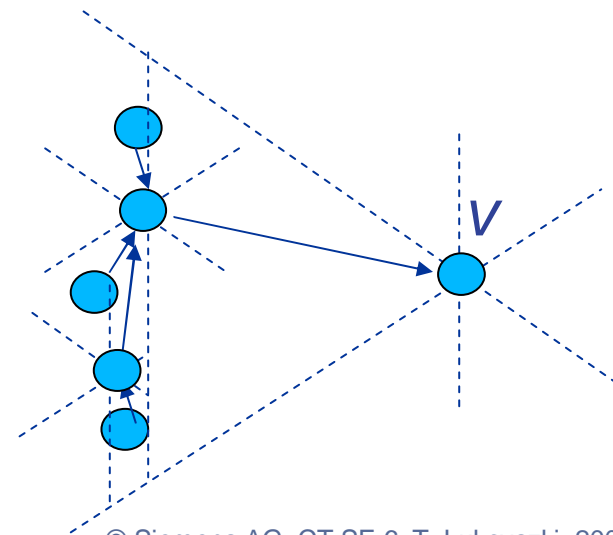
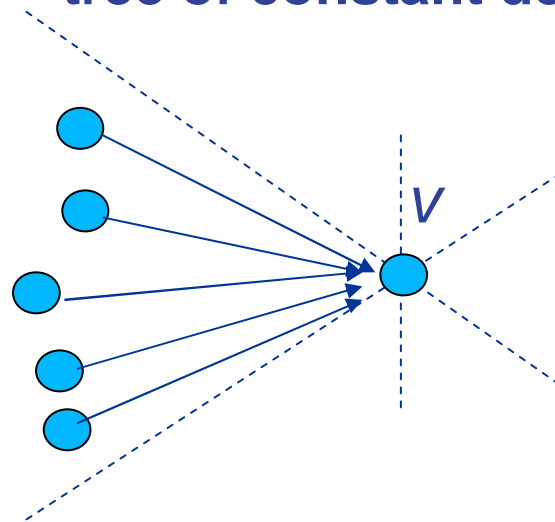
$$E := \{(u, v) \mid \forall w \neq v : \text{sector}(u, v) = \text{sector}(u, w) \Rightarrow D(u, v) < D(u, w)\}$$

Network Topologies

Let G_Y be the Yao graph.

The **Bounded Degree Yao graph (BoundY)** [Arya et al. 95] is defined by the following procedure:

- For each v in V and for each sector around v do
 - $N(v) := \{ w \mid (w,v) \in E(G_Y) \text{ and } w \in \text{sector}(v) \}$
 - Replace the star $\{ (w,v) \mid w \in N(v) \}$ by a certain tree of constant degree

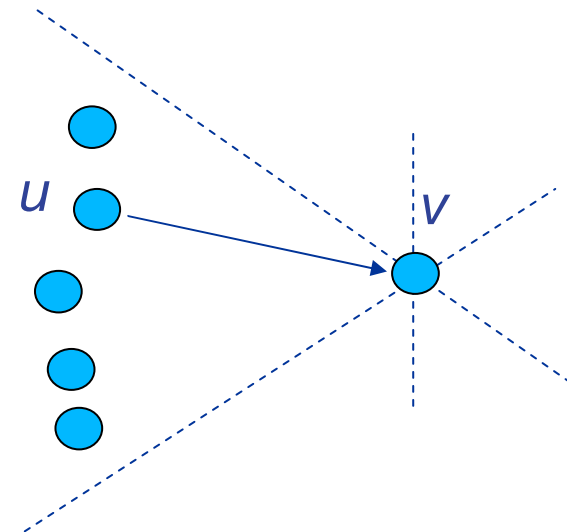
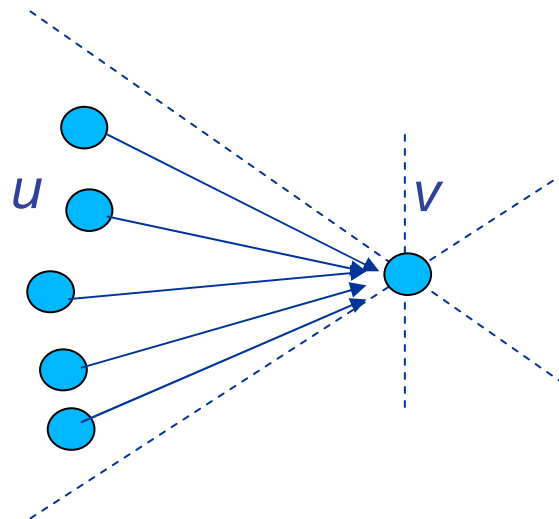


Network Topologies

Let G_Y be the Yao graph.

The Sparsified Yao graph (SparsY) [Li et al. 01] is defined by the following set of directed edges:

$$E := \{ (u,v) \in E(G_Y) \mid \forall w : (w,v) \in E(G_Y) \text{ and } \text{sector}(v,w) = \text{sector}(v,u) \Rightarrow D(v,u) < D(v,w) \}$$

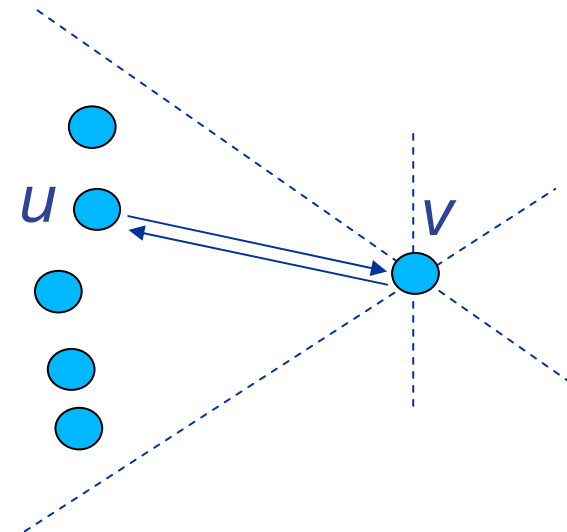
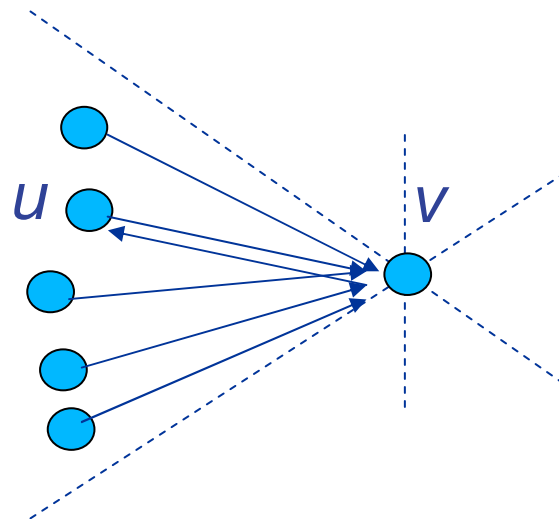


Network Topologies

Let G_Y be the Yao graph.

The Symmetric Yao graph (SymmY) [Li et al. 01] is defined by the following set of directed edges:

$$E := \{ (u,v) \in E(G_Y) \mid (v,u) \in E(G_Y) \}$$

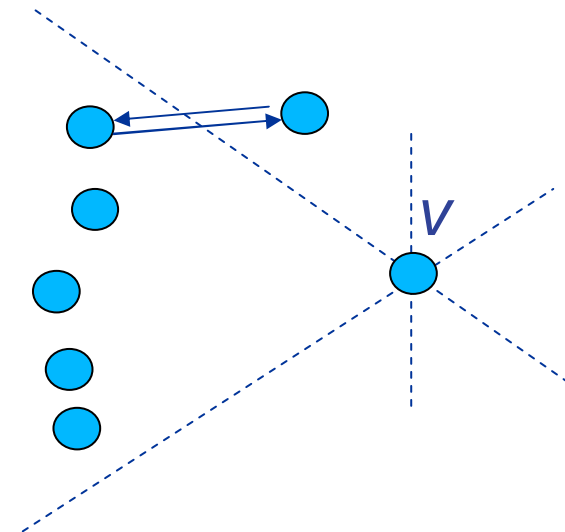
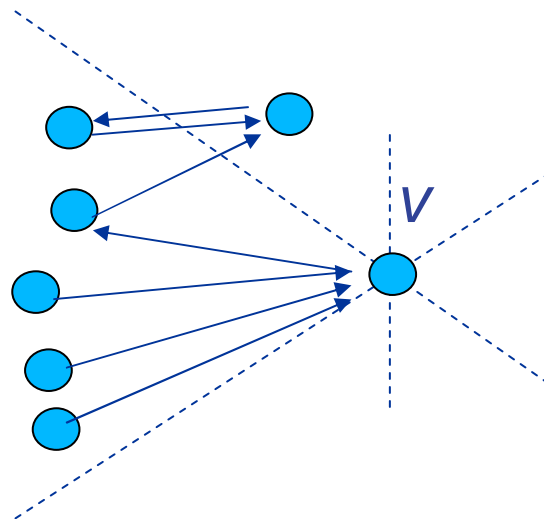


Network Topologies

Let G_Y be the Yao graph.

The **Symmetric Yao graph (SymmY)** [Li et al. 01] is defined by the following set of directed edges:

$$E := \{ (u,v) \in E(G_Y) \mid (v,u) \in E(G_Y) \}$$



Network Topologies

The Hierarchical Layer graph* (LH) is defined by the union of w graphs (layers) L_0, \dots, L_w

$$V = V(L_0) \supseteq V(L_1) \supseteq \dots \supseteq V(L_w) = \{v_0\}$$

Properties of $V(L_i)$:

Let $\alpha \geq \beta > 1$, $r_0 \leq \min_{u,v \in V} \|u,v\|_2$, $r_i := \beta^i r_0$

- Minimum distance:

$$\forall L_i \forall u,v \in V(L_i) : \|u,v\|_2 \geq r_i$$

- Covering:

$$\forall u \in V(L_i) \exists v \in V(L_{i+1}) : \|u,v\|_2 \leq r_{i+1}$$

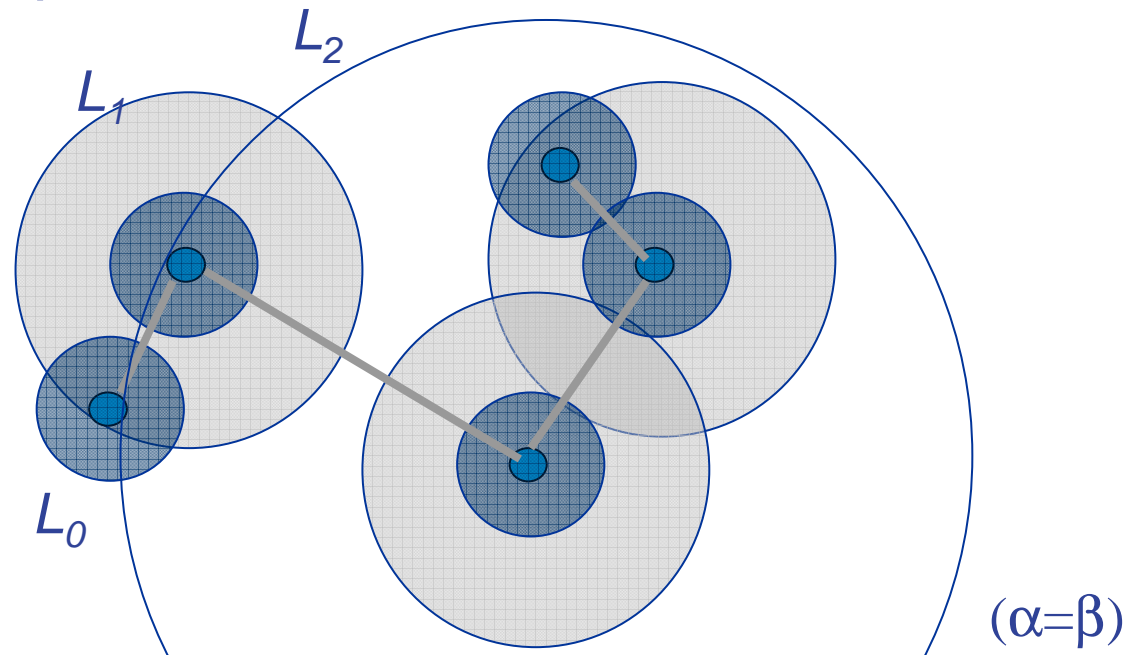
Edge set:

$$E(L_i) := \{ (u,v) \mid u,v \in V(L_i) \text{ and } \|u,v\|_2 \leq \alpha r_i \}$$



Network Topologies

layers in the HL graph for normal a vertex set is
 $w = O(\log n)$



Graph Properties

$$\text{SymmY}(V) \subseteq \text{SparsY}(V) \subseteq \text{Yao}(V)$$

$$\text{SparsY}(V) \subseteq \text{BoundY}(V).$$

Let V be a nice vertex set. Then

Topology	Yao	BoundY	SparsY	SymmY	HL
in-degree	$n-1$	$(k+1)^2$	k	k	$O(\log n)$
out-degree	k	k	k	k	$O(\log n)$
degree	$n-1+k$	$(k+1)^2+k$	$2k$	k	$O(\log n)$



Graph Properties

Definitions: Let $G=(V,E)$ be a graph

- G is a **c -spanner**, if $\forall u,v \in V \exists$ path P from u to v with
$$\|P\|_2 := \sum_{e \in P} \|e\|_2 \leq c \|u,v\|_2$$
- G is a **weak c -spanner**, if $\forall u,v \in V \exists$ path P from u to v which is covered by a disk of radius $c \|u,v\|_2$ centered at u
- G is a **(c,d) -power spanner**, if $\forall u,v \in V \exists$ path $P = (u=u_1, \dots, u_m=v)$ from u to v such that

$$\sum_{i=1}^{m-1} (\|u_i, u_{i+1}\|_2)^d \leq c \cdot \min_{(u=v_1, \dots, v_m=v)} \sum_{i=1}^{m-1} (\|u_i, u_{i+1}\|_2)^d$$

- G is a **power spanner**, if for all $d > 1$ there exists a constant c such that G is a (c,d) -power spanner.



Graph Properties

Let V be a set of n points in \mathbb{R}^2 . Then Yao(V) is

- a c -spanner for $k > 6$ with [Ruppert & Seidel 91]

$$c = \frac{1}{1 - 2 \sin(\Theta / 2)}$$

- a weak c -spanner for $k \geq 6$ with [Fischer et al. 97]

$$c = \max\left\{\sqrt{1 + 48 \sin^4(\Theta / 2)}, \sqrt{5 - \cos\Theta}\right\}$$

- a weak c -spanner for $k = 4$ with [Fischer et al. 98] *

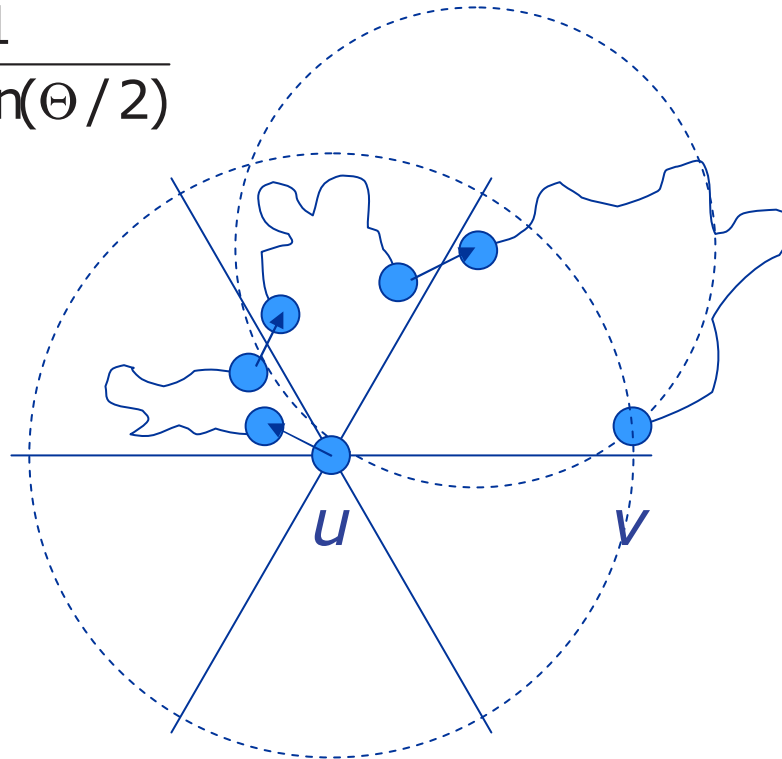
$$c = \sqrt{3 + \sqrt{5}}$$



Graph Properties

Theorem*: Let V is a set of n points in \mathbb{R}^2 . Then, $\text{SparsY}(V)$ is a weak c -spanner for $k > 6$ with

$$c = \frac{1}{1 - 2 \sin(\Theta / 2)}$$



Graph Properties

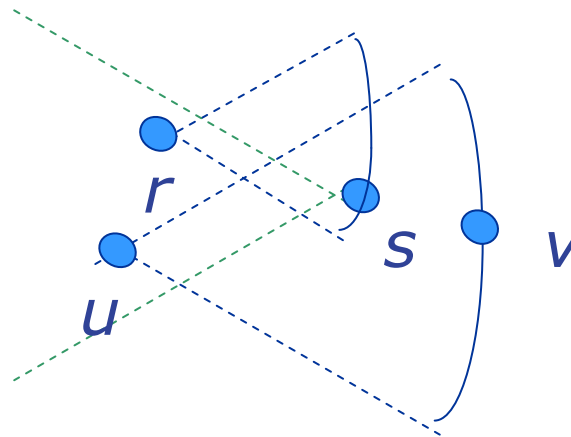
Theorem*: Let V be a set of n points in \mathbb{R}^2 and let $\alpha > 2\beta / (\beta - 1)$. Then $HL(V)$ is a c -spanner with

$$c = \max \left\{ \beta \frac{\alpha(\beta - 1) + 2\beta}{\alpha(\beta - 1) - 2\beta}, \frac{\alpha}{\beta} \right\}$$



Communication Features

- An edge (r,s) has an **uni-directional interference** caused by (u,v) , if $\text{sector}(s,r) = \text{sector}(s,u)$ and $\text{sector}(u,s) = \text{sector}(u,v)$ and $D(u,v) \geq D(u,s)$.



- An edge (r,s) has a **bi-directional interference** caused by (u,v) , if (r,s) has an uni-directional interference caused by (u,v) or if (u,v) has an uni-directional interference caused by (r,s) .



Communication Features

Let P be a path system for V , $E(P)$ the set of edges in P , and $l(e)$ the load on an edge e .

Congestion $C_P(V)$ and energy $E_P(V)$:

$$C_P(V) := \max_{e \in E(P)} \left\{ l(e) + \sum_{e' \in \text{Int}(e)} l(e') \right\}$$

$$E_P(V) := \sum_{e \in E(P)} l(e) \cdot \|e\|_2^2$$



Communication Features

Theorem*: Let G be a c -spanner. Then G is a (c^d, d) -power spanner.

Theorem: [Meyer a.d.H. et al. 02]: If for a *normal* vertex set a graph G is a weak c -spanner with uni-directional interference number q , then there is a path system in G which approximates the congestion optimal path system by a factor of $O(q \log n)$

- A vertex set is *normal*, if for a fixed polynomial $p(n)$:

$$\frac{\max_{u,v \in V} \|u,v\|_2}{\min_{u,v \in V} \|u,v\|_2} \leq p(n)$$



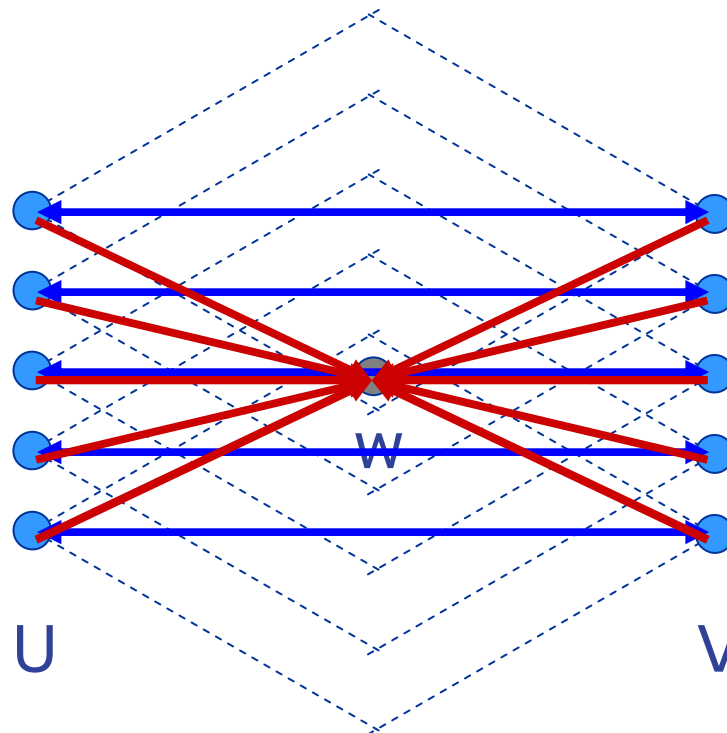
Communication Features

Theorem*: For a normal vertex set V the following table describes the worst case behavior, whether it hosts optimal path systems approximating energy or congestion.

Topology	uni-directional interference number	spanner	Energy approx. factor	Congestion approx. factor
Yao	$n-1$	yes	$O(1)$	--
BoundY	$\Theta(n)$	yes	$O(1)$	--
SparsY	1	weak and power	$O(1)$	$O(\log n)$
SymmY	1 (also bi-dir.!)	no, but connected	--	--
HL	$O(\log n)$	yes	$O(1)$	$O(\log^2 n)$

Maintaining the Network

Theorem*: Let V be a normal and nicely located vertex set. Then $\Theta(|V|)$ edges need to be changed, if an enter or leave operation takes place in $Yao(V)$, $BoundY(V)$, $SparsY(V)$ or $SymmY(V)$. For $HL(V)$ this number is $O(\log|V|)$.



Maintaining the Network

Theorem*: Let V be a normal and nicely located vertex set and m the number of involved edges. Then the network structure of $\text{Yao}(V)$, $\text{BoundY}(V)$, $\text{SparsY}(V)$ and $\text{SymmY}(V)$ can be rebuilt in $O(m \log s)$ time. For $\text{HL}(V)$ we need $O(\log |V| + \log s)$ time.

For $\text{Yao}(V)$:

enter

- Inform the nodes that u has entered
- Search for next neighbor of u in each sector
- The informed nodes check empty sectors, whether u can be its nearest neighbor
- The informed nodes check not-empty sectors, whether u is closer

leave

- At some time the node v notices that u has left the network
- v informs other nodes that u has left
- All nodes adjacent to u have to determine new neighbors (this is done by a reduced version of the enter-algorithm)

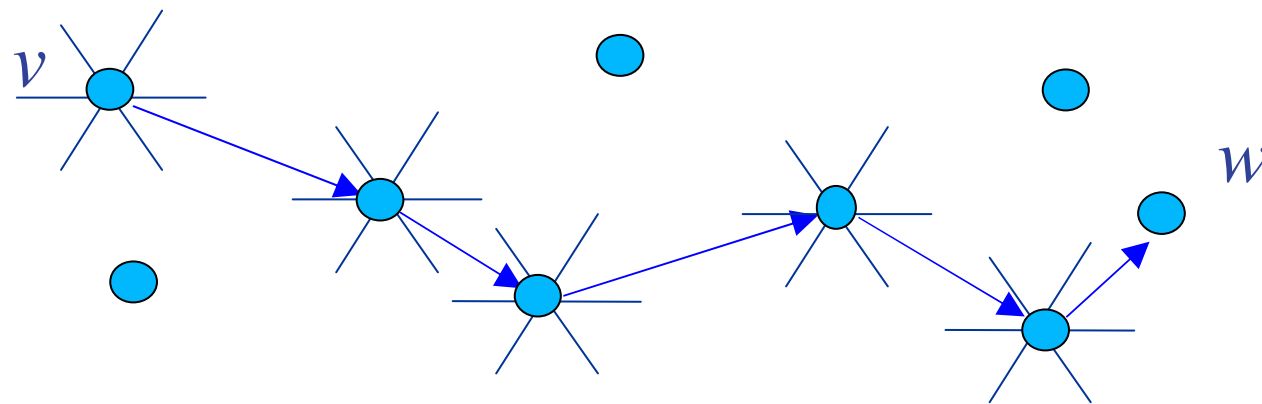
Summary – Topology Control

Topology	Congestion approx. factor	Energy approx. factor	enter/leave involved nodes	Time for enter/leave
Yao	--	$O(1)$	$\Theta(n)$	$O(n \log s)$
BoundY	--	$O(1)$	$\Theta(n)$	$O(n \log s)$
SparsY	$O(\log n)$	$O(1)$	$\Theta(n)$	$O(n \log s)$
SymmY	--	--	$\Theta(n)$	$O(n \log s)$
HL	$O(\log^2 n)$	$O(1)$	$O(\log n)$	$O(\log n + \log s)$



Position Based Ad-Hoc Routing

- Each node knows its position (GPS)
- Example: The Yao-graph
- It allows a memoryless position based routing
- Forward the package to the neighbor in the sector which contains the destination node
- But how can the source discover the position of the destination?



Hypercube Location Service (HLS)*

Theorem*: HLS has the properties:

- **Storage requirement per node:** $\Theta(\log n)$
with high probability (w.h.p.), i.e. $\text{Prob} \geq 1 - n^{-\alpha}$.
 - independent from the geographic distribution of the nodes
- **Fault tolerance:** Even if a constant fraction of the nodes fails simultaneously, a packet will be delivered w.h.p.
- **Update costs, when a node enters or leaves:**
 - # affected nodes: $O(\log n)$, w.h.p.
 - time at an affected node: $O(\log n)$, w.h.p.



Summary

- **Focus:**
 - I Topology Control**
 - II Position Based Routing**
 - III Distributed Location Services**
- **Immediate network level support for position based services**
- **Cost efficient and highly reliable self-organizing (and hybrid) multihop networks for future networking scenarios**



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